

University of Alberta Library



0 1620 3416174 3

For Reference

NOT TO BE TAKEN FROM THIS ROOM

Ex LIBRIS
UNIVERSITATIS
ALBERTAENSIS



A REPORT OF AN
INVESTIGATION OF
ELECTROSMOSIS

S. R. Hayden



Digitized by the Internet Archive
in 2018 with funding from
University of Alberta Libraries

CONTENTS

| | |
|---|------------|
| General | 1 |
| Theoretical Considerations | 3 |
| Soil Properties | 11 |
| Apparatus and Test Procedure | 11 |
| Diagram of Electosmometer | 12 |
| Photograph of Electosmometer | 13 |
| Determination of k | 14 |
| Determination of k_e | 14 |
| Voltage vs. Hydraulic Gradient | 15 |
| Discussion of Experimental Results | |
| Constant k | 16 |
| Constant k_e | 17 |
| Voltage vs. Hydraulic Gradient | 21 |
| Comparison of Hydrostatic to Osmotic Flow | 23 |
| Summary of Observations and Conclusions | 25 |
| Bibliography | 28 |
| Appendix | |
| Plot of Volume of Water Transferred under Hydrostatic Head vs. Time for BC Clay | Graph 1 |
| for Calgary Silt | Graph 3 |
| Plot of Volume of Water Transferred in One Hour vs Hydrostatic Head for BC Clay | Graph 2 |
| for Calgary Silt | Graph 4 |
| Plot of Hydraulic Permeability vs Absolute Pressure | Graph 5 |
| Plot of Volume of Water Transferred Under Potential Gradients vs Time for BC Clay | Graph 6 |
| for Calgary Silt | Graph 7, 8 |

| | |
|---|--------------|
| Plot of Discharge vs. Potential Gradients, for Calgary Silt | Graph 9 |
| Plot of k_e vs Potential Gradients | Graph 10 |
| Plot of Absolute Pressure vs. Potential Gradients for BC Clay | Graph 11 |
| for Calgary Silt | Graph 12 |
| Plot of Pressure Rise vs. Time for BC Clay | Graph 13 |
| for Calgary Silt | Graph 14, 15 |
| Plot of Current vs Time for Calgary Silt | Graph 16,17 |

PREFACE

The following report deals with an investigation on electrosmosis carried out by the author during the winter of 1948-49. The greatest part of time was consumed constructing an apparatus and developing a technique which was suitable to obtain the hydraulic and electric permeabilities and the relationship between voltage gradient and hydraulic head. The soils used were rather different in their physical properties - one being a silt, the other a clay.

Various observations are reported that have been made during the course of the tests and recommendations for further testing have been made. Difficulties encountered and errors made have also been included so that other investigators do not have to spend time needlessly.

The author wishes to thank Professor J. A. Harle for his interest in and direction of the investigation. Thanks are also due to Mr. R.W. King, B.Sc., whose able assistance in construction details was freely given and greatly appreciated.

S.R. Hayden

April 1949

GENERAL

Electro-osmosis, one of the electro-kinetic phenomena, is a means of moving a fluid through a media. This is applicable to the draining of fine grained soils into a well point where normal methods of ground water lowering are not satisfactory. The forces involved may be employed in a reverse manner also, that is they may be used as a potential barrier opposing the normal drainage thus keeping ground water away from some particular area.

The initial phenomenon was first described by Reuss (2) in 1808 with subsequent experimental contributions by Wiedmann (3) and by Quincke (4). Theoretical development is due mainly to Helmholtz who is responsible for the electric double-layer concept.

The action of electro-osmosis (or electro-endosmosis) has been proposed to be as follows:-

Consider a capillary tube filled with a fluid. Due to an existing ionic condition in the fluid, the inner wall of the capillary becomes coated with an electrical double layer, one charge is on the liquid in immediate contact with the capillary and the other on the capillary. Now, if an external potential difference is applied to the capillary it will cause the movable inner layer to be displaced relative to the

capillary, its motion depending upon the sign of the electrical charge. In pure water the wall, for a great majority of solids, is charged negatively with respect to the water and, therefore, the water moves towards the negative pole.

THEORETICAL CONSIDERATIONS

Consider a capillary filled with water.

Empirically it has been found that the water in such a capillary moves under electric potential. Thus, it must be assumed that the cylindrical surface of the liquid carries a charge which must be equal and opposite to one carried by the capillary wall.

Let the charge/cm length of surface of the liquid cylinder be "e" and the applied electric field "E". Then we have a force acting on the cylinder of :-

$$F = eE$$

The liquid solid system represents a cylindrical condenser with capacity:-

$$C = \frac{DS}{4\pi d} \text{ where}$$

D = dielectric constant of water,

S = surface area of condenser,

d = distance between charges.

If the capillary is assumed to be of unit length and to have a circular cross section of radius "r". then:-

$$S = 2\pi r$$

$$C = \frac{2\pi r D}{4\pi d} = \frac{rD}{2d}$$

The first of the season was a very fine one, and the weather was very pleasant. The wind was from the north, and the sea was very calm. The temperature was about 60 degrees, and the sun was shining brightly. The water was very clear, and the fish were very plentiful. The birds were also very numerous, and the air was very fresh. The day was very pleasant, and the weather was very good. The wind was from the north, and the sea was very calm. The temperature was about 60 degrees, and the sun was shining brightly. The water was very clear, and the fish were very plentiful. The birds were also very numerous, and the air was very fresh.

The second of the season was also a very fine one, and the weather was very pleasant. The wind was from the north, and the sea was very calm. The temperature was about 60 degrees, and the sun was shining brightly. The water was very clear, and the fish were very plentiful. The birds were also very numerous, and the air was very fresh. The day was very pleasant, and the weather was very good. The wind was from the north, and the sea was very calm. The temperature was about 60 degrees, and the sun was shining brightly. The water was very clear, and the fish were very plentiful. The birds were also very numerous, and the air was very fresh.

Let ψ be the electric potential between adjoining charged surfaces, then:-

$$\psi = \frac{e}{C} = \frac{e2d}{rD}$$

$$\text{or } e = \frac{rD\psi}{2d}$$

substituting back in equation(1), we have:-

$$F_e = eE = \frac{rDE\psi}{2d}$$

The movement of the liquid is resisted by the viscosity of the liquid " η ". This force F_r can be expressed as:-

$$F_r = S \eta \frac{\Delta v}{\Delta r} \quad \text{where}$$

S = surface area of cylinder of 1 cm.

length = $2\pi r$

$\frac{\Delta v}{\Delta r}$ = rate of velocity change with increasing distance from the capillary wall.

If we assume $\frac{\Delta v}{\Delta r} = \text{constant} = \frac{v}{d}$, we get a steady flow condition where the forces are equal. This assumption is valid only for a range of values of r as will be shown later.

$$F_e = F_r$$

$$\text{or } \frac{rDE\psi}{2d} = \frac{2\pi r\eta v}{d}$$

Consider the function $f(x) = \sin(x)$ on the interval $[0, 2\pi]$.

Find the maximum and minimum values of $f(x)$ on $[0, 2\pi]$.

$$f'(x) = \cos(x)$$

$$f'(x) = 0 \Rightarrow \cos(x) = 0$$

Solve for x in the interval $[0, 2\pi]$ where $\cos(x) = 0$.

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

Also check the values of $f(x)$ at the endpoints $x = 0$ and $x = 2\pi$.

Calculate $f(0)$, $f(\frac{\pi}{2})$, $f(\frac{3\pi}{2})$, and $f(2\pi)$.

Compare these values to find the maximum and minimum.

$$f(0) = 0, f(\frac{\pi}{2}) = 1, f(\frac{3\pi}{2}) = -1, f(2\pi) = 0$$

Therefore, the maximum value is 1 and the minimum value is -1.

Answer: Maximum = 1, Minimum = -1

Find the area under the curve $y = \sin(x)$ from $x = 0$ to $x = \pi$.

Set up the definite integral for the area.

$$\text{Area} = \int_0^{\pi} \sin(x) dx$$

Integrate $\sin(x)$ with respect to x from 0 to π .

The antiderivative of $\sin(x)$ is $-\cos(x)$. Evaluate from 0 to π .

Calculate $-\cos(\pi) - (-\cos(0))$.

The area under the curve is 2.

$$\text{Area} = 2$$

$$\int_0^{\pi} \sin(x) dx = 2$$

$$v = \frac{DE\psi}{4\pi\eta}$$

or if V = volume of water transferred per
unit time

$$V = \pi r^2 v = \frac{\pi r^2 ED\psi}{4\pi\eta} = \frac{r^2 D\psi E}{4\eta}$$

From this the following results are:-

- (1) Amount of water transferred is proportional to the square of the radius, of the capillary (within limits), the applied potential and a factor $\frac{D\psi}{4\eta}$
- (2) Since ψ is proportional to e , the amount of liquid moved is directly proportional to the quantity of electricity conducted. This is Wiedemann's Law.

We can consider the factor $\frac{D\psi}{4\eta}$ as a characteristic constant of the solid-liquid interfacial system. Since D and η are statistical averages, dependant upon temperature, and ψ is always associated with D and η , it is best to let $\frac{D\psi}{4\eta} = k_0$.

We now have:- $V = r^2 k_0 E$

Now, if we let one square cm. of the capillary system contain n_c capillaries, then the volume flow is,

$$Vn_c = r^2 k_0 E n_c,$$

however, since the porosity of the system is equal to

$$n = r^2 \pi n_c$$

we have

$$Vn_c = k_0 n E$$

and allowing

$$V_p = V_{nc}, \quad k = k_0 \frac{\pi}{n}$$

Let $f(x) = x^2 + 2x + 1$ and $g(x) = x^2 - 2x + 1$.

Find $f(g(x))$.

$$f(g(x)) = (x^2 - 2x + 1)^2 + 2(x^2 - 2x + 1) + 1$$

Expand the expression and simplify.

First, expand $(x^2 - 2x + 1)^2$ using the binomial theorem:

$(x^2 - 2x + 1)^2 = (x^2 - 2x + 1)(x^2 - 2x + 1)$

$$= x^4 - 2x^3 + x^2 - 2x^3 + 4x^2 - 2x + x^2 - 2x + 1$$

Combine like terms:

$$= x^4 - 4x^3 + 6x^2 - 4x + 1$$

Now, add $2(x^2 - 2x + 1) + 1$:

$$= x^4 - 4x^3 + 6x^2 - 4x + 1 + 2x^2 - 4x + 2 + 1$$

Combine like terms again:

$$= x^4 - 4x^3 + 8x^2 - 8x + 4$$

Therefore, $f(g(x)) = x^4 - 4x^3 + 8x^2 - 8x + 4$.

$$f(g(x)) = x^4 - 4x^3 + 8x^2 - 8x + 4$$

Final answer: $x^4 - 4x^3 + 8x^2 - 8x + 4$

Let $f(x) = x^2 + 2x + 1$ and $g(x) = x^2 - 2x + 1$.

Find $f(g(x))$.

$$f(g(x)) = (x^2 - 2x + 1)^2 + 2(x^2 - 2x + 1) + 1$$

Expand the expression and simplify.

$$f(g(x)) = x^4 - 4x^3 + 8x^2 - 8x + 4$$

we have $V_p = knE$, or for $nk = k_e$

$$V_p = k_e E .$$

This expression is the same as that derived by Darcy for the flow under hydrostatic head. If we say that the flow due to hydrostatic head is $V_H = kI$ where k is the coefficient of permeability, and I is the hydraulic gradient, the total velocity of pore water resulting from the summation of electrostatic and hydrostatic heads will be given by

$$\begin{aligned} \underline{V_t} &= \underline{V_p} + \underline{V_H} \\ &= \underline{k_e E} + \underline{kI} \end{aligned}$$

From this equation, we may compute the rise in piezometric head with time. If we say,

- F = cross section of piezometric tube
- A = cross section of the sample
- h = height of rise at constant level at time t after starting the test
- k = coefficient of permeability in cm/sec.
- k_e = coefficient of electrical permeability in cm^2 per volt sec.
- U = electric potential applied to electrodes in volts.
- d = distance between electrodes in cms.

then, during any time dt , the piezometric rise will be dh and the flow through the sample Fdh .

The electric gradient causes a flow of: -

$$\begin{aligned} dV_p &= k_e E A dt \\ &= k_e \frac{U}{d} A dt \end{aligned}$$

The hydrostatic gradient causes a flow of: -

$$\begin{aligned} dV_h &= k I A dt \\ &= k \frac{h}{d} A dt \end{aligned}$$

The flow equation becomes: -

$$\begin{aligned} Fdh &= dV_p - dV_h \\ &= k_e \frac{U}{d} A dt - k \frac{h}{d} A dt \end{aligned}$$

$$\frac{dh}{dt} = \frac{(k_e U - kh)}{Fd} A$$

or $\frac{dh}{k_e U - kh} = \frac{A dt}{Fd}$, integrating, we get

$$-\frac{1}{k} \ln (k_e U - kh) = \frac{A}{Fd} t + C_1$$

$$\text{at } t = 0 \quad h = 0$$

$$\therefore C_1 = -\frac{\ln k_e U}{k}$$

$$\frac{\ln (k_e U - kh)}{k_e U} = -\frac{k A t}{Fd}$$

$$\frac{k_e U - kh}{k_e U} = e^{-\frac{k A t}{Fd}}$$

(Continued next page)

... ..

... ..

$$\Delta V = \int \mathbf{E} \cdot d\mathbf{l}$$

$$\Delta V = \int \frac{\rho}{\epsilon_0} d\mathbf{l}$$

... ..

$$\Delta V = \int \frac{\rho}{\epsilon_0} d\mathbf{l}$$

$$\Delta V = \int \frac{\rho}{\epsilon_0} d\mathbf{l}$$

... ..

$$\Delta V = \int \frac{\rho}{\epsilon_0} d\mathbf{l}$$

$$\Delta V = \int \frac{\rho}{\epsilon_0} d\mathbf{l}$$

$$\Delta V = \int \frac{\rho}{\epsilon_0} d\mathbf{l}$$

$$\Delta V = \int \frac{\rho}{\epsilon_0} d\mathbf{l} \quad (1)$$

$$\Delta V = \int \frac{\rho}{\epsilon_0} d\mathbf{l}$$

$$\Delta V = \int \frac{\rho}{\epsilon_0} d\mathbf{l}$$

$$\Delta V = \int \frac{\rho}{\epsilon_0} d\mathbf{l}$$

$$\Delta V = \int \frac{\rho}{\epsilon_0} d\mathbf{l}$$

$$\Delta V = \int \frac{\rho}{\epsilon_0} d\mathbf{l}$$

$$\begin{aligned}
 1 - \frac{kh}{k_e U} &= e^{-\frac{kAt}{Fd}} \\
 k_e &= \frac{kh}{U} \cdot \frac{1}{\left(1 - e^{-\frac{kAt}{Fd}}\right)} \times \frac{e^{-\frac{kAt}{Fd}}}{e^{-\frac{kAt}{Fd}}} \\
 &= \frac{kh}{U} \times \frac{e^{-\frac{kAt}{Fd}}}{\left(e^{-\frac{kAt}{Fd}} - 1\right)}
 \end{aligned}$$

This is the general solution to the electro-osmotic conditions. In our case Fdh is very small and the equation simplifies to: -

$$k_e = \frac{kh}{U}$$

It might be well to observe that in the derivation of these equations, some very important influences have been neglected. These are: -

(1) Electrolysis occurs at the cathode due to the galvanic current in the capillaries.

(2) Conductivity of the soil particles was neglected.

A more accurate consideration would be as given by Smoluchowski (6). This is as follows: - the current is composed of a galvanic phase, a surface current transporting the charges of a double layer, a third phase flowing through the particles. Thus, the form of the velocity

$$\frac{1}{x^2} = x^{-2}$$

$$\frac{d}{dx} x^{-2} = -2x^{-3} = -\frac{2}{x^3}$$

$$= -\frac{2}{x^3}$$

Let $y = x^{-2}$. Then $\frac{dy}{dx} = -2x^{-3} = -\frac{2}{x^3}$.
 Let $y = x^{-3}$. Then $\frac{dy}{dx} = -3x^{-4} = -\frac{3}{x^4}$.
 Let $y = x^{-4}$. Then $\frac{dy}{dx} = -4x^{-5} = -\frac{4}{x^5}$.
 Let $y = x^{-5}$. Then $\frac{dy}{dx} = -5x^{-6} = -\frac{5}{x^6}$.

Let $y = x^{-6}$. Then $\frac{dy}{dx} = -6x^{-7} = -\frac{6}{x^7}$.
 Let $y = x^{-7}$. Then $\frac{dy}{dx} = -7x^{-8} = -\frac{7}{x^8}$.
 Let $y = x^{-8}$. Then $\frac{dy}{dx} = -8x^{-9} = -\frac{8}{x^9}$.
 Let $y = x^{-9}$. Then $\frac{dy}{dx} = -9x^{-10} = -\frac{9}{x^{10}}$.
 Let $y = x^{-10}$. Then $\frac{dy}{dx} = -10x^{-11} = -\frac{10}{x^{11}}$.
 Let $y = x^{-11}$. Then $\frac{dy}{dx} = -11x^{-12} = -\frac{11}{x^{12}}$.
 Let $y = x^{-12}$. Then $\frac{dy}{dx} = -12x^{-13} = -\frac{12}{x^{13}}$.

Let $y = x^{-13}$. Then $\frac{dy}{dx} = -13x^{-14} = -\frac{13}{x^{14}}$.
 Let $y = x^{-14}$. Then $\frac{dy}{dx} = -14x^{-15} = -\frac{14}{x^{15}}$.
 Let $y = x^{-15}$. Then $\frac{dy}{dx} = -15x^{-16} = -\frac{15}{x^{16}}$.
 Let $y = x^{-16}$. Then $\frac{dy}{dx} = -16x^{-17} = -\frac{16}{x^{17}}$.
 Let $y = x^{-17}$. Then $\frac{dy}{dx} = -17x^{-18} = -\frac{17}{x^{18}}$.
 Let $y = x^{-18}$. Then $\frac{dy}{dx} = -18x^{-19} = -\frac{18}{x^{19}}$.
 Let $y = x^{-19}$. Then $\frac{dy}{dx} = -19x^{-20} = -\frac{19}{x^{20}}$.

distribution in the capillaries is not yet clear and will have to be found by physical researches.

In the laboratory it has been found that the effect of electrosmotic disturbances is considerable. During the experiment polarization of the electrodes and increases in electrical resistance have been observed. It has also been observed by others that accessory phenomena influenced osmotic rise (6). These influences would be less evident in the field due to a large scale application, whereas in the laboratory, they are of considerable importance. We should, therefore, apply values found in the beginning of the experiment that have not been influenced very much by accessory phenomena. If we consider the equation

$$k_e = \frac{kh}{U} \frac{\epsilon \frac{kAt}{Fd}}{(\epsilon \frac{kAt}{Fd} - 1)}$$

This may be written: -

$$\begin{aligned} h &= \frac{k_e U}{k} \frac{\epsilon \frac{kAt}{Fd} - 1}{\epsilon \frac{kAt}{Fd}} \quad \text{or} \\ &= \frac{k_e U}{k} \left(1 - \epsilon \frac{-kAt}{Fd} \right) \end{aligned}$$

the first part of the paper, the author has shown that the
 conditions of the problem are satisfied by the function

$$u(x, y) = \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{1 - \cos \theta} d\theta \quad (1)$$

which is the solution of the problem. The author then shows
 that the function (1) is the only solution of the problem.

The author then shows that the function (1) is the only
 solution of the problem. The author then shows that the function (1)

is the only solution of the problem. The author then shows that the function (1)

is the only solution of the problem. The author then shows that the function (1)

is the only solution of the problem. The author then shows that the function (1)

is the only solution of the problem. The author then shows that the function (1)

$$u(x, y) = \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{1 - \cos \theta} d\theta \quad (1)$$

is the only solution of the problem.

$$u(x, y) = \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{1 - \cos \theta} d\theta \quad (1)$$

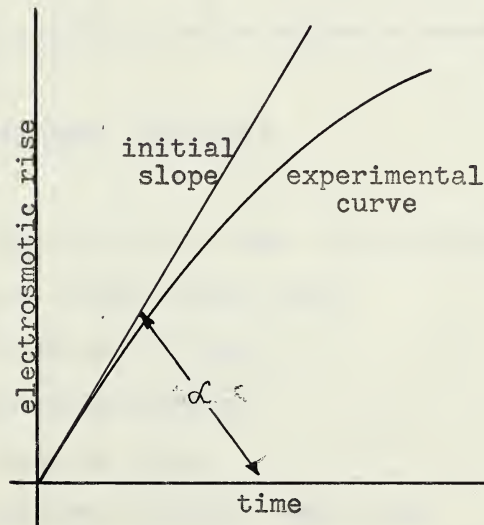
$$u(x, y) = \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{1 - \cos \theta} d\theta \quad (1)$$

To get the velocity of pressure rise, we differentiate the above and get: -

$$\frac{dh}{dt} = \frac{k_e U}{k} \cdot \frac{kA}{Fd} \cdot e^{-\frac{kAt}{Fd}} \text{ and at } t = 0$$

$$\left. \frac{dh}{dt} \right|_{t=0} = k_e \frac{U}{d} \frac{A}{F} = \tan \alpha \text{ where } \alpha \text{ is the angle as shown below.}$$

$$k_e = \frac{Fd}{AU} \tan \alpha$$



EXPERIMENTAL WORK in ELECTROSMOSIS

The work undertaken by the author consisted of developing a technique for the determination of the hydraulic and electric permeabilities k and k_e and obtaining a relationship between potential gradient and hydrostatic head. The soils used were Calgary silt (from the mudslide on the North Hill) and B.C. Clay (Fairmont). Their properties are shown below.

| | Shrinkage Limit | Plastic Limit | Liquid Limit | Sp.Gr. | Plasticity Index | Flow Index | Standard Proctor Compaction | Coeff. of Consol. |
|-----------------|--------------------|------------------|-----------------|--------|---------------------|---------------|-----------------------------------|-------------------------|
| Calgary Silt | 15.3 | 18.9 | 24.8 | 2.69 | 5.9 | 7.5 | 111.4#/ft ³ @ 14.4% | |
| BC Clay | | | | | | | | |

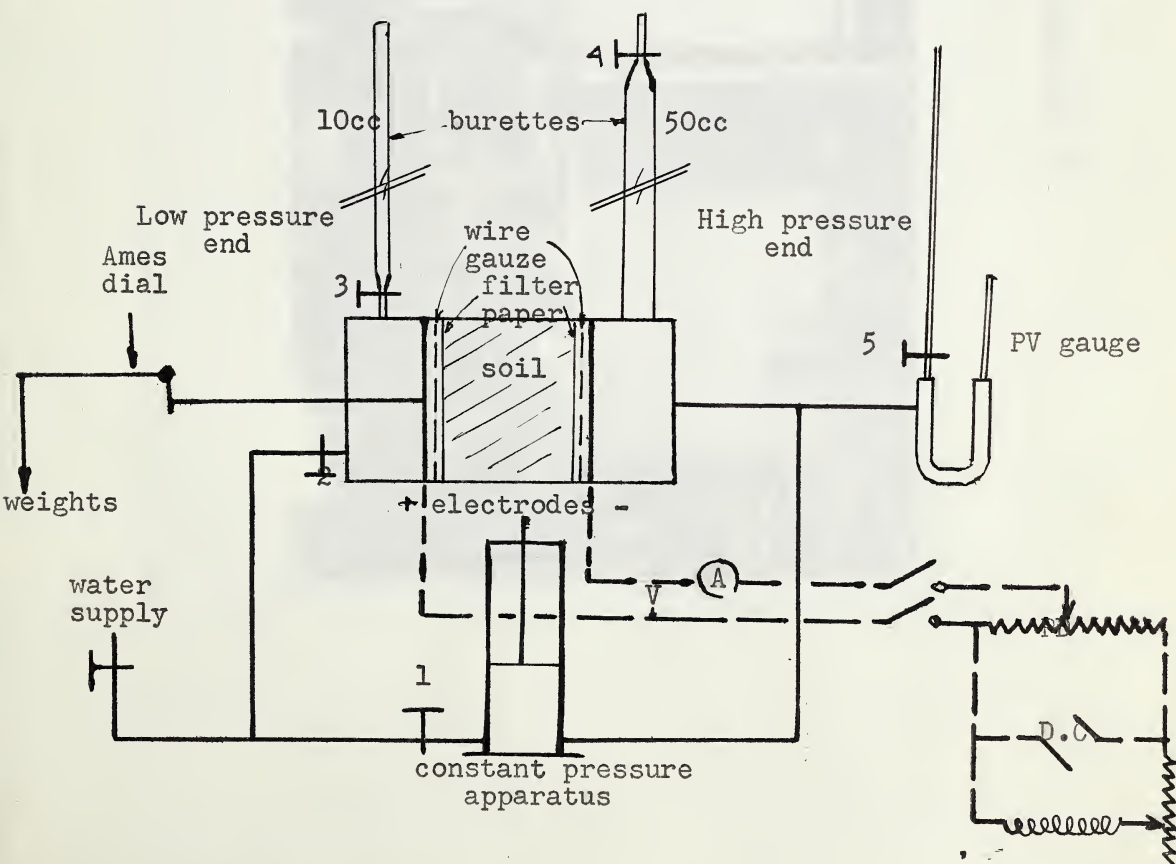
Apparatus and Test Procedure

The following items of equipment were employed:

- (1) D.C. Power supply unit
- (2) Potential Divider
- (3) Electrosometer
- (4) pressure gauge
- (5) constant pressure apparatus.

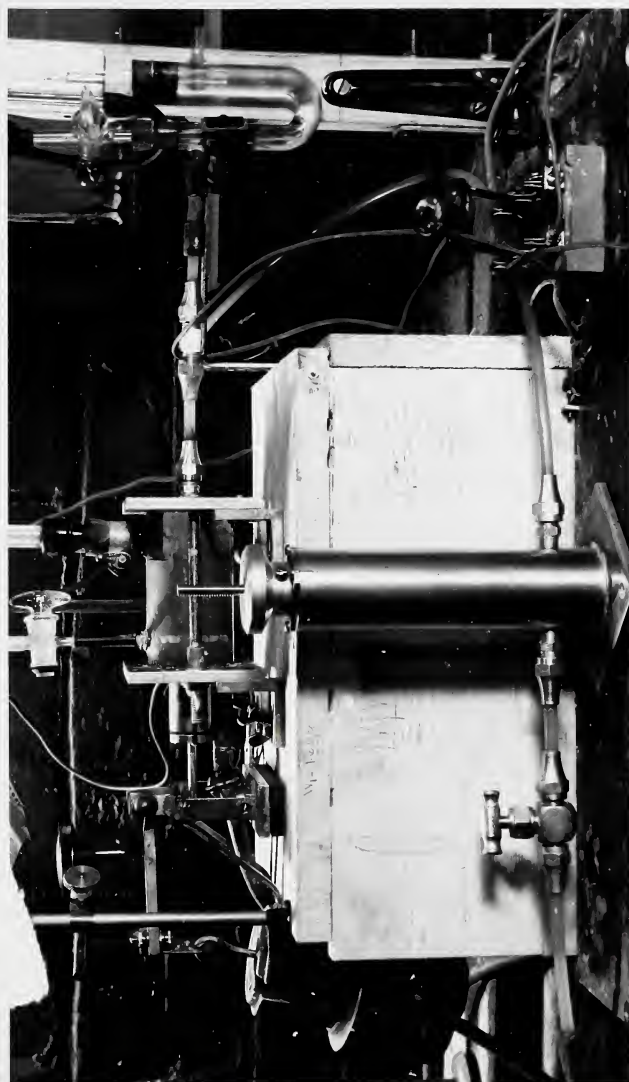
The electrosometer is much the same as the one described by Gueze, de Bruyn and Joustra (5) but, with a few modifications, was made more versatile. It consists

of a plastic cylinder 9.3 cms. long, 4.4 cms. inside diameter and .3 cms. thick, fitted with aluminum end plates. Iron electrodes were used because it is expected that this material would be used in the field. The plastic cylinder was drilled to accomodate two burettes, one at the high pressure end and the other at the low pressure end. The burette at the high pressure end had to be fitted upside down so that the hydrogen that was liberated at the cathode might have some way to escape. The following diagram and photograph are given to present a clear picture of the apparatus as it was employed.



The first part of the paper is devoted to a discussion of the
 various methods which have been proposed for the determination of
 the rate of reaction between a solid and a liquid. It is shown that
 the most reliable method is that of measuring the change in weight
 of the solid as the reaction proceeds. This method is applicable to
 all cases in which the solid is insoluble in the liquid. It is
 also applicable to cases in which the solid is soluble in the liquid,
 provided that the solid is in the form of a powder or of small
 pieces. The method of measuring the change in weight of the solid
 is described in detail, and it is shown that it is possible to
 determine the rate of reaction with a high degree of accuracy.
 The second part of the paper is devoted to a discussion of the
 various factors which influence the rate of reaction between a solid
 and a liquid. It is shown that the rate of reaction is influenced
 by the nature of the solid, by the nature of the liquid, by the
 temperature, and by the surface area of the solid. The effect of
 each of these factors is discussed in detail, and it is shown that
 the rate of reaction can be increased by increasing the surface area
 of the solid, by increasing the temperature, and by using a liquid
 which is more reactive than the one which is being used.





AN ELECTROSMOMETER

Determination of "k"

To determine the hydraulic permeability "k" the soil sample was packed and tamped into the cylinder, using a wire gauze spacer and a filter paper at each end to keep the soil from seeping into the holes in the electrodes. The apparatus was assembled as shown above and water introduced from the supply to the low pressure end (valves 1 and 5 closed). Secondly, water was introduced into the high pressure end (valves 2 and 5 closed). Next, valves 1 and 4 were closed and valve 5 opened so as to obtain a reading of atmospheric pressure on the gauge, then valve 5 was closed.

By means of the constant pressure apparatus (consisting of a brass cylinder with a screw adjusted piston) any pressure up to 2,200 cms. of water absolute was applied to the high pressure end of the osmometer. At constant time intervals the amount of water transferred was observed by reading the burette on the low pressure end. A constant pressure was maintained by continuous adjustments of the screw on the constant pressure apparatus.

Pressure was read on the PV gauge which was calibrated with a monometer.

Determination of " k_e "

As before, the sample was packed and both ends

of the cylinder were filled with water. This time, however, it was not necessary to use the constant pressure apparatus, so, it was removed and in its place a graduate was used to measure the amount of water passed through by an electrical potential gradient. A potential was applied across the electrodes and, at constant time intervals, the amount of water transferred was observed.

This was repeated for both soils for various potential gradients.

Determination of Potential Gradient versus Hydrostatic Head

As in part one, the sample was packed and the apparatus assembled. A potential was applied across the electrodes and then the hydrostatic head was adjusted by means of the constant pressure apparatus so that no water transfer occurred in either direction. This condition was observed on the low pressure end burette which could be read to .01 cc.

In all tests the samples represented 100 grams dry weight of Calgary silt and B.C. Clay. Each sample was mixed with an amount of tap water approximately equal to the liquid limit of the respective soils. Also in all tests of part one, the hydraulic head was kept constant while in all tests of part two, the potential gradient was kept constant. In part three the potential gradient was

kept constant and the hydraulic head adjusted to obtain a balance.

DISCUSSION of EXPERIMENTAL RESULTS

In the following the results of the experiments will be discussed with specific reference to the graphs that are included.

Constant k

The hydraulic permeability "k" was determined as given above. Graphs 1 and 3 show the linear relation between water transferred versus time for constant pressure, while graphs 2 and 4 show the linearity between water transferred versus absolute pressure for unit time.

In all the tests the major difficulty encountered was that of leakage between the sample and the cylinder. In graph 1, the 2,140 cm line shows the effect of such a leakage. These leakages must be watched for or they may cause much confusion in the interpretation of the test results. Calgary silt could not be tested for pressures above 1,850 cms of water absolute because the soil would be forced from the sample into the low pressure chamber and leakage would result.

The coefficient of hydraulic permeability
in centimeters per second for B.C. Clay was found to be:

| | | | | | | |
|-----------------------|---------|----|------|-----|----------|------|
| 1.46×10^{-7} | cms/sec | at | 1000 | cms | of water | abs. |
| 1.46×10^{-7} | " | " | 1150 | " | " | " |
| 1.36×10^{-7} | " | " | 1365 | " | " | " |
| 1.30×10^{-7} | " | " | 1680 | " | " | " |
| 1.30×10^{-7} | " | " | 2140 | " | " | " |

These values, except the last, were obtained by testing one sample. The variation is seen to be quite small, about 5% of the mean value of 1.38×10^{-7} cms/sec.

For Calgary silt the following values were obtained:

| | | | | | | |
|-----------------------|---------|----|------|-----|------------|---|
| $.985 \times 10^{-6}$ | cms/sec | at | 1840 | cms | for sample | 1 |
| $.948 \times 10^{-6}$ | " | " | 1660 | " | " | 1 |
| $.861 \times 10^{-6}$ | " | " | 1510 | " | " | 1 |
| $.477 \times 10^{-6}$ | " | " | 1490 | " | " | 2 |
| $.470 \times 10^{-6}$ | " | " | 1230 | " | " | 2 |
| $.366 \times 10^{-6}$ | " | " | 1050 | " | " | 2 |

It can be seen that there is a small variation in each sample and a larger one between samples. It was found, however, that the mean value, $.69 \times 10^{-6}$ cms/sec gave satisfactory results as is shown on graph 11.

Constant "k_e"

The coefficient of electrical permeability "k_e" in centimeters² per volt second was obtained as described above. Graphs 6, 7 and 8 show water transferred in cubic centimeters against time in minutes for various voltage gradients. Graph 6 shows very well the difficulty in obtaining values that may be checked in the laboratory. The 4 volt /cm lines show linearity with time for constant

Table 1. Summary of the data for the 1990s. The data are presented in the following format:

| Year | Country | Population (millions) | GDP (billions of dollars) | Life expectancy (years) | Infant mortality rate (per 1,000 live births) |
|------|---------|-----------------------|---------------------------|-------------------------|---|
| 1990 | USA | 248 | 5,800 | 75.4 | 10.6 |
| 1991 | USA | 249 | 5,900 | 75.5 | 10.5 |
| 1992 | USA | 250 | 6,000 | 75.6 | 10.4 |
| 1993 | USA | 251 | 6,100 | 75.7 | 10.3 |
| 1994 | USA | 252 | 6,200 | 75.8 | 10.2 |

Source: U.S. Census Bureau, Bureau of Economic Analysis, and Centers for Disease Control and Prevention.

Note: The data are presented in the following format: Year, Country, Population (millions), GDP (billions of dollars), Life expectancy (years), and Infant mortality rate (per 1,000 live births).

The data are presented in the following format: Year, Country, Population (millions), GDP (billions of dollars), Life expectancy (years), and Infant mortality rate (per 1,000 live births).

The data are presented in the following format: Year, Country, Population (millions), GDP (billions of dollars), Life expectancy (years), and Infant mortality rate (per 1,000 live births).

| Year | Country | Population (millions) | GDP (billions of dollars) | Life expectancy (years) | Infant mortality rate (per 1,000 live births) |
|------|---------|-----------------------|---------------------------|-------------------------|---|
| 1995 | USA | 253 | 6,300 | 75.9 | 10.1 |
| 1996 | USA | 254 | 6,400 | 76.0 | 10.0 |
| 1997 | USA | 255 | 6,500 | 76.1 | 9.9 |
| 1998 | USA | 256 | 6,600 | 76.2 | 9.8 |
| 1999 | USA | 257 | 6,700 | 76.3 | 9.7 |

The data are presented in the following format: Year, Country, Population (millions), GDP (billions of dollars), Life expectancy (years), and Infant mortality rate (per 1,000 live births).

The data are presented in the following format: Year, Country, Population (millions), GDP (billions of dollars), Life expectancy (years), and Infant mortality rate (per 1,000 live births).

The data are presented in the following format: Year, Country, Population (millions), GDP (billions of dollars), Life expectancy (years), and Infant mortality rate (per 1,000 live births).

The data are presented in the following format: Year, Country, Population (millions), GDP (billions of dollars), Life expectancy (years), and Infant mortality rate (per 1,000 live births).

Table 1. Summary of the data for the 1990s.

The data are presented in the following format: Year, Country, Population (millions), GDP (billions of dollars), Life expectancy (years), and Infant mortality rate (per 1,000 live births).

The data are presented in the following format: Year, Country, Population (millions), GDP (billions of dollars), Life expectancy (years), and Infant mortality rate (per 1,000 live births).

The data are presented in the following format: Year, Country, Population (millions), GDP (billions of dollars), Life expectancy (years), and Infant mortality rate (per 1,000 live births).

The data are presented in the following format: Year, Country, Population (millions), GDP (billions of dollars), Life expectancy (years), and Infant mortality rate (per 1,000 live births).

The data are presented in the following format: Year, Country, Population (millions), GDP (billions of dollars), Life expectancy (years), and Infant mortality rate (per 1,000 live births).

The data are presented in the following format: Year, Country, Population (millions), GDP (billions of dollars), Life expectancy (years), and Infant mortality rate (per 1,000 live births).

The data are presented in the following format: Year, Country, Population (millions), GDP (billions of dollars), Life expectancy (years), and Infant mortality rate (per 1,000 live births).

voltage gradients, however, there seems to be no constancy about k_e . This may be explained by two variables in the experiment. The first is the degree of tamping given the samples - this could not be controlled. This would affect the effective value of the capillary radius which changes the flow by a squared factor (within limits) as shown in the theory. The second variable, which was controlled to some extent, is the degree of plasticity of the sample. The three samples of the 4 volt/cm line were mixed differently. All had 150 gms of soil but the first was mixed with 75 cc of water, the second with 50 cc and the third with 25 cc. This shows that k_e is considerably changed by the amount of water in the samples, being greater with more water.

Graph 7 is for Calgary silt. It shows three interesting results in the testing. The first observation is that the amount of water collected was greater than that supplied in all tests. This means that the sample was being dried faster than water entering it. This was substantiated by the observation that the cathode end of the soil was dry and hard at the end of the tests while the anode was still wet.

The second observation was that there was a marked decrease in k_e when the sample was undisturbed and tested again. This is shown by the change of slope between the lines 1 and 2, 3 and 4. This indicated that the properties of the soil have been altered but, whether this is actually the case or just a result of using small samples

will have to be checked in the field. One factor affecting " k_e " in the laboratory tests will be the $\text{Fe}(\text{OH})_3$ that is formed at the anode. The $\text{Fe}(\text{OH})_3$ is observed as a blue-green stain moving through the sample. Upon drying, the compound breaks down and may be observed as a reddish stain, Fe_2O_3 .



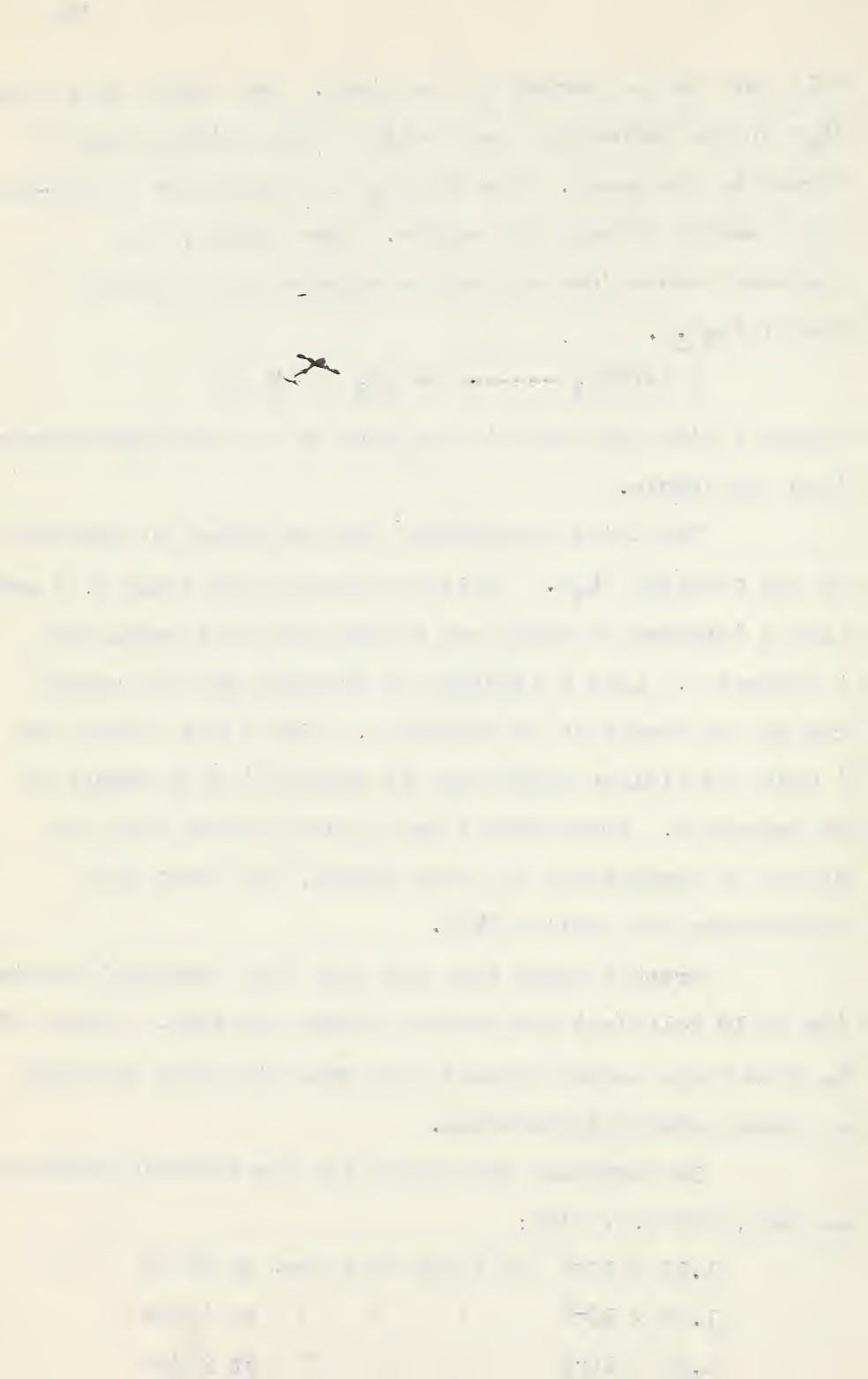
During a five hour test it may move up to three centimeters into the sample.

The third observation was the effect of temperature on the constant " k_e ". This is shown by the lines 5, 6 and 7. Line 5 (plotted by dots) was a first run on a sample at 4 degrees C. Line 6 (plotted by crosses) was the second run on the sample at 20 degrees C. Line 7 was a first run (under as similar conditions as possible) on a sample at 20 degrees C. From these lines it can be seen that the effect of temperature is quite marked, the lower the temperature the smaller " k_e ".

Graph 8 shows that for very high potential gradients (up to 10 volts/cm) the results remain the same. Values of k_e from these curves checked very well with that obtained at lower potential gradients.

In computing the values for the electric permeability we have, for B.C. Clay:

| | | | |
|-------------------------|-----------------|---------------|----------|
| 1.12 x 10 ⁻⁵ | cm ² | per volt sec. | at 6V/cm |
| 1.58 x 10 ⁻⁵ | " | " | at 4V/cm |
| 1.22 x 10 ⁻⁵ | " | " | at 2V/cm |



This shows that there is a maximum value for " k_e " at 4 volts/cm. The mean value is $1.3 \times 10^{-5} \text{ cm}^2$ per volt sec. Higher voltages could not be used on B.C. Clay since the sample shrank, leakage resulted and the test terminated.

For Calgary silt there seems to be a linear relationship between the flow in centimeters per second and voltage gradient. This is shown on graph 9.

The values computed for " k_e " are:

| | |
|-----------------------|--|
| 1.14×10^{-5} | cms^2 / volt sec at 11.4 V/cm |
| 1.03×10^{-5} | " " at 8.6 V/cm |
| $.87 \times 10^{-5}$ | |
| $.70 \times 10^{-5}$ | |
| 1.14×10^{-5} | " " at 4.3 V/cm |
| 1.28×10^{-5} | " " at 2.9 V/cm |

This shows that there is a minimum value for " k_e " at 8.6 V/cm. The mean value is $1.0 \times 10^{-5} \text{ cm}^2$ / volt sec. Graph 10 shows the variation of " k_e " with the voltage gradient.

In obtaining " k_e ", the initial difficulty was in removing the hydrogen from the cathode. This was solved by using a 50 cc burette upside down connected to the high pressure end of the electrosmometer. This gave the hydrogen a large area to escape through. Smaller openings were found to slow up the hydrogen, causing and undesirable accumulation.

Relation Between Potential Gradient and Hydrostatic Head

Graphs 11 and 12 show the hydrostatic head that may be maintained by a potential gradient. The experimental and theoretical results check very closely, at least over the practical range of voltage gradients.

Graph 11, for B.C. Clay, indicates a secondary phase in the relation between the gradients. It has been suggested that the higher voltages cause the clay to collapse on itself and seal its pores. Shrinkage of the sample was observed during the test, an observation that gives some support to the suggestion.

Graph 12, for Calgary silt, gives very good agreement between theoretical and observed results. Since the pressure gauge could not be read closer than 20 cms. of water at high pressures, the results are well within the experimental error.

In the derived expression for the relation between gradients we have:

$$h = \frac{k_e}{k} U, \text{ which for B.C. Clay becomes}$$

$$h = \frac{1.3 \times 10^{-5} U}{1.38 \times 10^{-7}}$$

$$= 94 U.$$

For Calgary silt we obtain by the same method

$$h = 14.5 U$$

We have derived in the theory the equation

$$k_e = \frac{Fd}{AU} \tan \alpha.$$

From this expression we should get the value of k_e at the beginning of the test. This equation was found to give results of the order of 100 times greater than the previous method. The reason for this variation probably lies in the fact that the expression was to be used with an electrosmometer with a large value of F . Our value of F was very small, about $.12 \text{ cm}^2$, which would invalidate the above expression for our particular apparatus.

Graphs 13 and 14 show pressure rise with time for B.C. Clay and Calgary silt respectively. $\tan \phi$ can be obtained from these graphs. Graph 14 gives the variation in the results for various tests. The brown and red lines give results of individual tests and the black line shows the mean value.

Graphs 15, 16 and 17 are of general interest only. Graph 15 is a pressure run for various voltage gradients (first and second tests on each sample) plotted for three hours. The main observation is that the second test gives a slow linear pressure rise while the first test gives a rapid rise that reaches a maximum and maintains a constant high pressure. The significance of this is not known. It is probable that the change in the soil characteristics is mainly due to the small soil sample.

Graph 16 and 17 show currents with time for constant potential gradients. It is observed that polarization affects

the current within two minutes. It is also observed that for the higher voltage gradients this effect seems to be overcome. The current rises again to nearly its former value, then decaying smoothly with time. It is not certain what causes the current to rise again but it may be due to the fact that the hydrogen begins to escape from the cathode. Graph 17 shows the variability of currents with the test sample under constant voltages. The colored lines are the different test results and the black line gives the mean value.

Comparison of Flow due to Hydrostatic Head and Osmotic Flow

To compare quantity flow it is necessary to use the same units. For convenience, the unit will be taken as cubic centimeters per hour per pressure gradient.

Hydraulic Flow

Units:

$$\begin{array}{ccc} \text{volume} & \text{area} & \text{gradient} \\ \frac{\text{cm}^3}{\text{hour}} & \frac{1}{\text{cm}^2} & \frac{\text{cm}}{\text{cm}} \\ & \text{cm/hour} & \end{array}$$

Osmotic Flow

Units:

$$\begin{array}{ccc} \text{volume} & \text{area} & \text{gradient} \\ \frac{\text{cm}^3}{\text{hour}} & \frac{1}{\text{cm}^2} & \frac{\text{cm}}{\text{Volt}} \\ & \text{cm}^2/\text{volt hour} & \end{array}$$

since the area is the same for each test, it has been cancelled out.

B.C. Clay

| | | | |
|---|---------------|------------------------|---------|
| 1.78 x 10 ⁻³ cc/hr per cm/cm | | 1.75 cc/hr per volt/cm | |
| | @2140 cms abs | | @ 6V/cm |
| 1.78 x 10 ⁻³ | @1680 cms abs | 1.75 | @4V/cm |
| 1.88 x 10 ⁻³ | @1365 cms abs | | |
| 2.25 x 10 ⁻³ | @1150 cms abs | 2.0 | @ 2V/cm |
| 2.0 x 10 ⁻³ | @1000 cms abs | | |

Calgary Silt

| | | | |
|-------------------------|---------------|------|---------|
| 1.35 x 10 ⁻² | @1840 cms abs | 1.07 | @ 6V/cm |
| 1.30 x 10 ⁻² | @1660 cms abs | | |
| 1.16 x 10 ⁻² | @1510 cms abs | 1.25 | @ 4V/cm |
| .65 x 10 ⁻² | @1487 cms abs | | |
| .64 x 10 ⁻² | @1230 cms abs | 1.6 | @ 2V/cm |
| .50 x 10 ⁻² | @1050 cms abs | | |

From this set of values it can be seen that the volume of water transferred in cc/hour per unit gradient is 1000 times greater for electrosmotic than hydraulic flow for B.C. Clay and from 100 to 200 times greater for Calgary silt.

SUMMARY of OBSERVATIONS and CONCLUSIONS

This research must be considered as a preliminary investigation and the conclusions as tentative. Many more tests on different soils must be made before final conclusions may be drawn.

- (1) Electrosmotic permeability of soil is not constant by varies with moisture content, temperature and, to a small extent, with the applied potential.
- (2) Hydraulic permeability was found to vary slightly with the amount of water passed through the sample and with the samples. It is known that it changes with temperature, liquid media, integrated total flow of liquid media, influencing interface. (7)
- (3) Passage of D.C. current consolidates the material; .4% for BC Clay in 17 hours and .6% for Calgary silt in 30mins.
- (4) Passage of D.C. current changes the physical properties of the material. Cathode drying was observed. This changes the potential gradient in the sample.
- (5) Passage of D.C. current increases the temperature due to resistance heating, thus changing the factor $\frac{D\phi}{4n}$.
- (6) There is marked deterioration of the iron anode due to the liberated oxygen (electrolysis) attacking it.
- (7) Quantity of hydrogen produced during tests was found to be within $\frac{1}{2}$ of 1% of that computed due to the amount of current flowing.

(8) It was observed that $\text{Fe}(\text{OH})_3$ was transferred as a blue green stain 13 centimeters into the sample during a five hour test. On drying, the hydroxide changed to an oxide that could be observed as a reddish stain.

(9) The quantity of water passed per coulomb of electricity was found to be .045 cc/coulomb. Wiedmann's law was obeyed.

(10) For practical purposes, the relation between hydrostatic head and potential ~~gradient~~ may be expressed as:

$$h = \frac{k_e}{k} U \quad *$$

where h = hydrostatic head in cms of water (gauge)
 k_e = electrosmotic permeability
 k = hydraulic permeability
 U = applied volts

(11) The values of k and k_e and the relationship between potential and hydrostatic head may be obtained by the electrometer. It is expected that the apparatus could be used to measure streaming potential and consolidation.

| | k_e | k | $h = \frac{k_e}{k} U$ |
|--------------|----------------------|-----------------------|-----------------------|
| B.C. Clay | 1.3×10^{-5} | 1.38×10^{-7} | $h = 94 U$ |
| Calgary Silt | 1.0×10^{-5} | 6.9×10^{-7} | $h = 14.5 U$ |

* It will be noted that the relation $h = \frac{k_e}{k} U$ is an expression relating potentials or gradients depending on the choice of units, i.e:

$$\frac{h}{l} = \frac{k_e}{k} \frac{U}{l}$$

where $\frac{h}{l}$ = hydraulic gradient and $\frac{U}{l}$ = voltage gradient

In conclusion it is suggested that an apparatus be built that is ribbed on the inside and that has probes fused into the plastic cylinder. The ribbing would reduce the effect of shrinkage of the sample and the probes would allow observation of the potential gradient during the tests.

It is also suggested that all further testing be done at low potential gradients, up to a maximum of two volts per centimeter. This is recommended since, although higher potential gradients are interesting, low potential gradients will be used in the field.

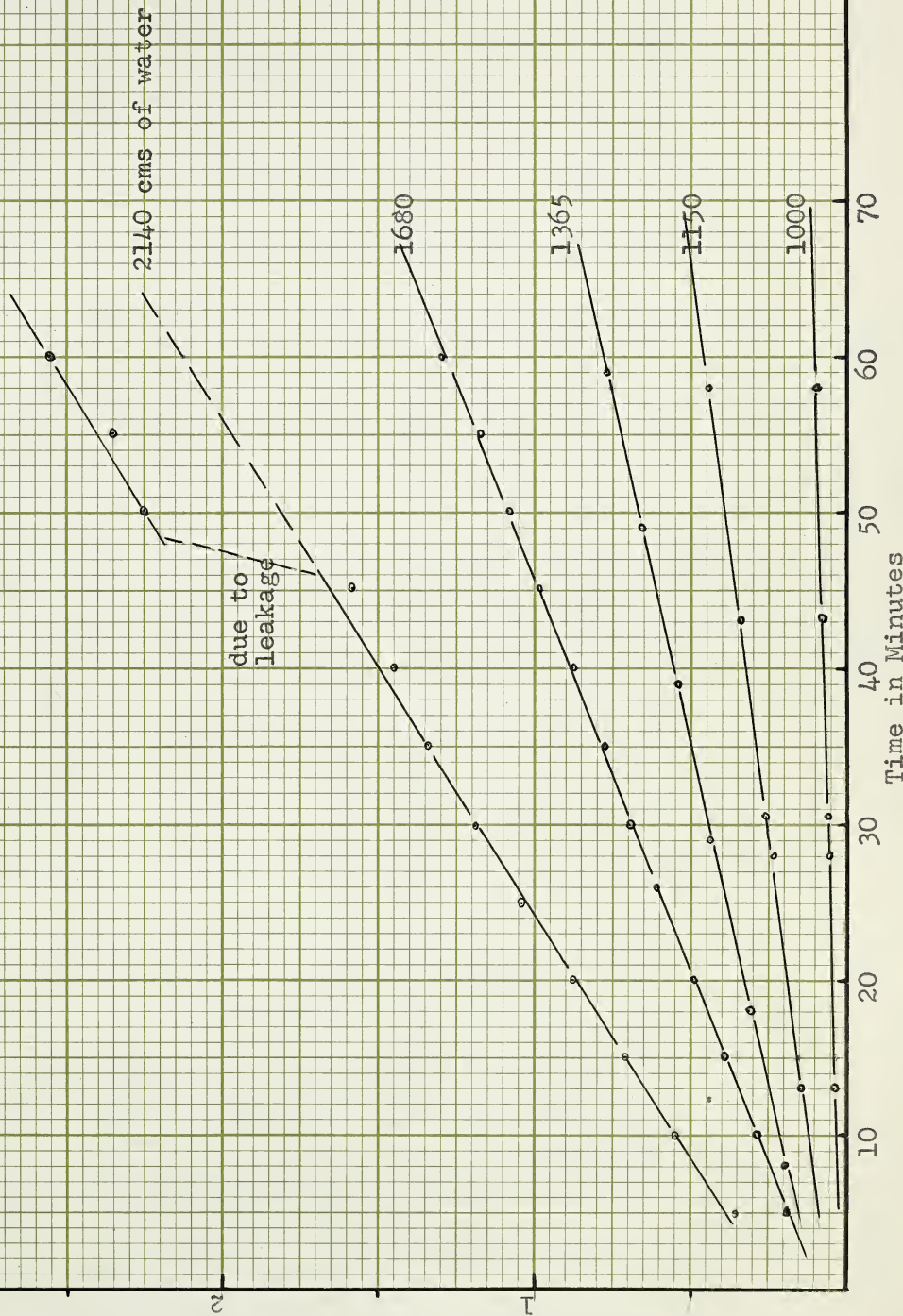
BIBLIOGRAPHY

- (1) Jour. Chim. Phys. 2,601, (1904); 3,50 (1905).
- (2) Reuss, Memoirs of the Imperial Russian Naturalist Society, Moscow, 2, 327-37.
- (3) Wiedmann, Pogg. Ann. 87, 327 (1852); 99, 177, (1856)
- (4) Quincke G, Pogg. Ann. 113, 513-598 (1861)
- (5) Proceedings of the Second International Conference of Soil Mechanics and Foundation Engineering, 1948
 - section 11 f 2
 - section 11 f 3
 - section 11 f 4
- (6) Proceedings of Twenty-Seventh Annual Meeting of The Highway Research Board 1947.

Newman, Electrolytic Conduction
Thompson, Electrochemistry
- (7) Terzaghi and Peck, Soil Mechanics in Engineering Practice

GRAPH #1.

PLOT of VOLUME of WATER TRANSFERRED
UNDER HYDROSTATIC HEAD v.s. TIME
B.C. CLAY

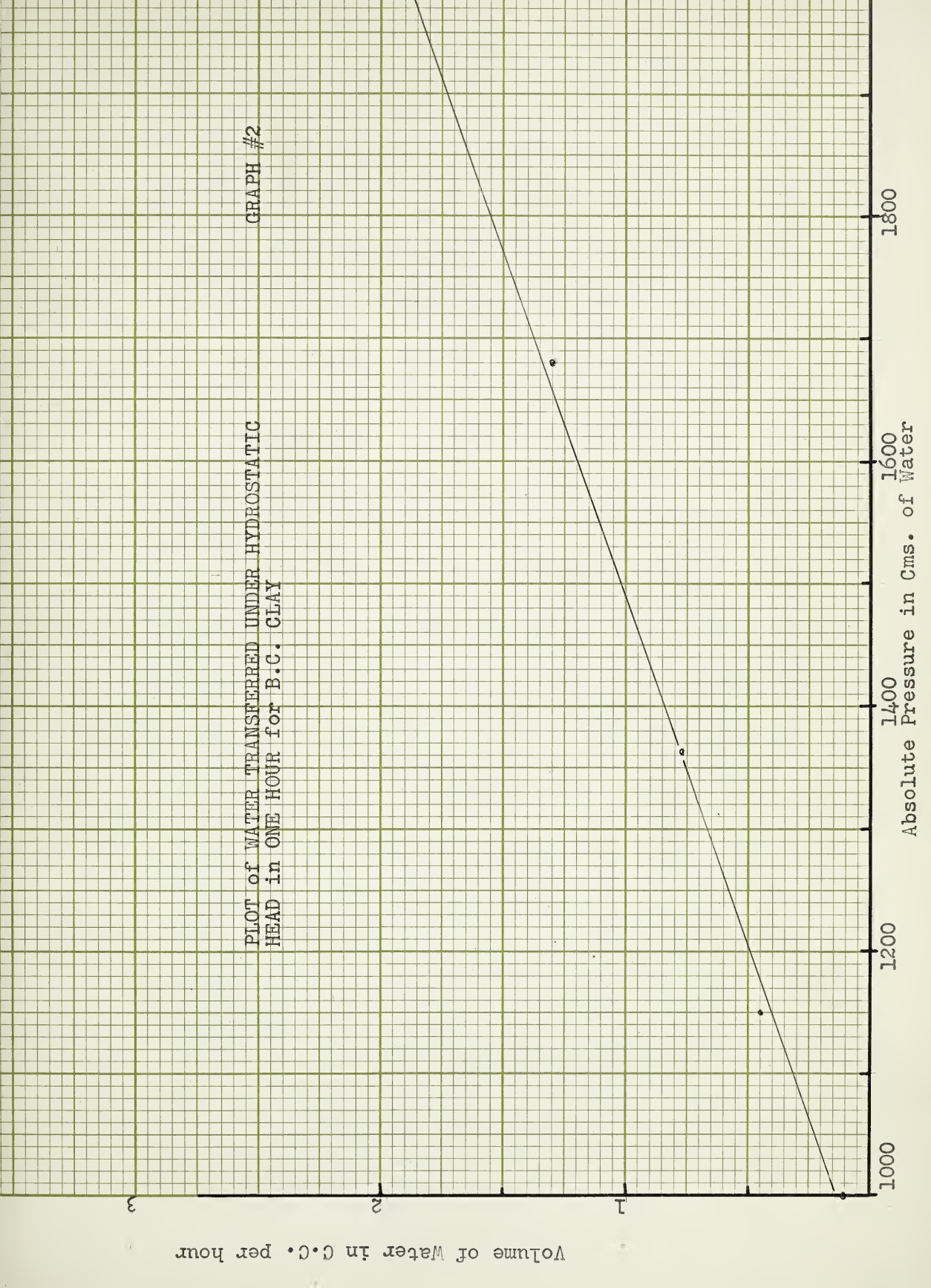


Volume of Water in C.C. per hour

PLOT of WATER TRANSFERRED UNDER HYDROSTATIC
HEAD in ONE HOUR for B.C. CLAY

GRAPH #2

1800
1600
1400
1200
1000
Absolute Pressure in Cms. of Water



1840 cms of water

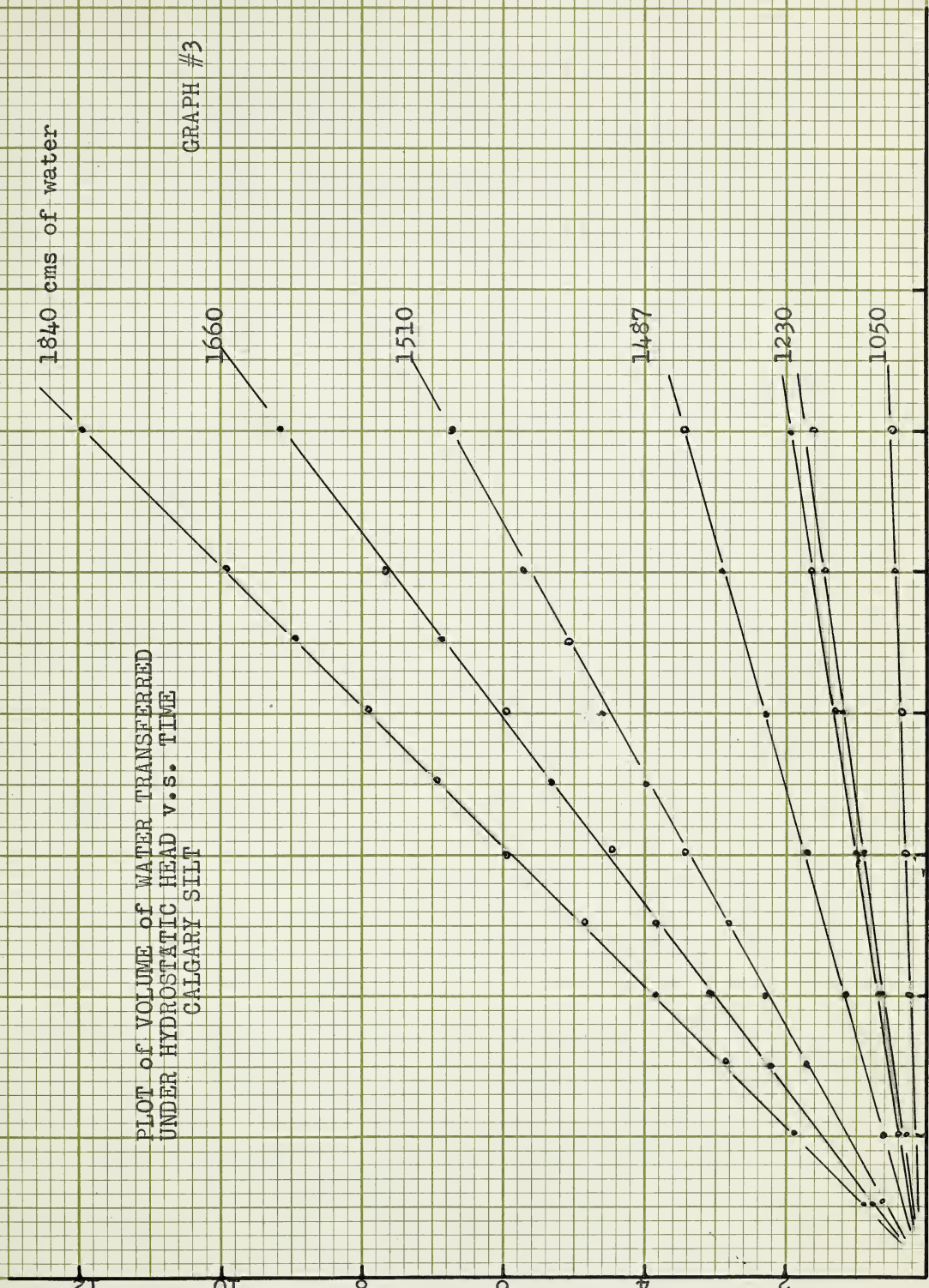
GRAPH #3

PLOT of VOLUME of WATER TRANSFERRED
UNDER HYDROSTATIC HEAD v.s. TIME
CALGARY SILT

1660 1510 1487 1230 1050

Time in Minutes

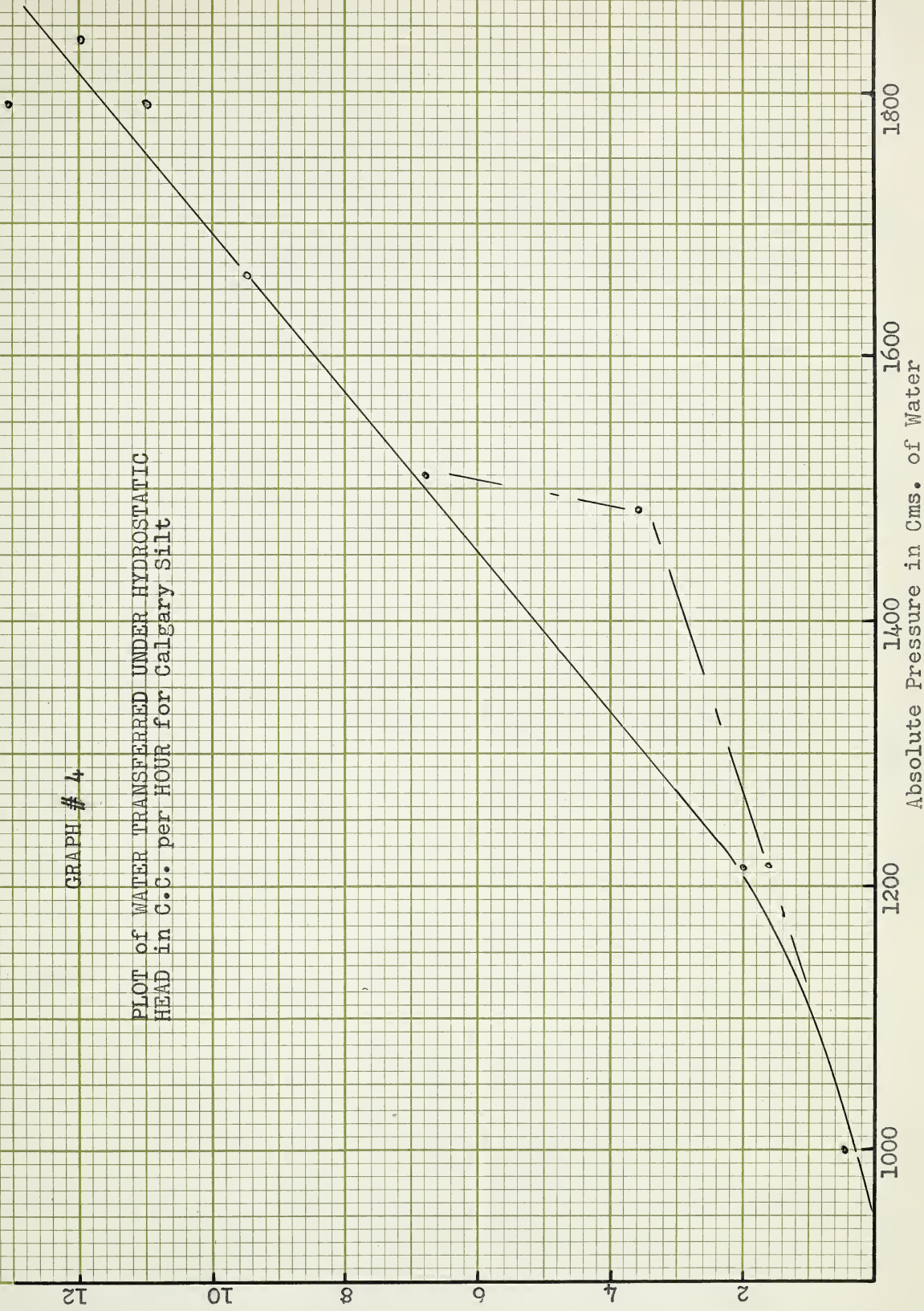
Volume in C.C.



Volume of Water in C.C. per Hour

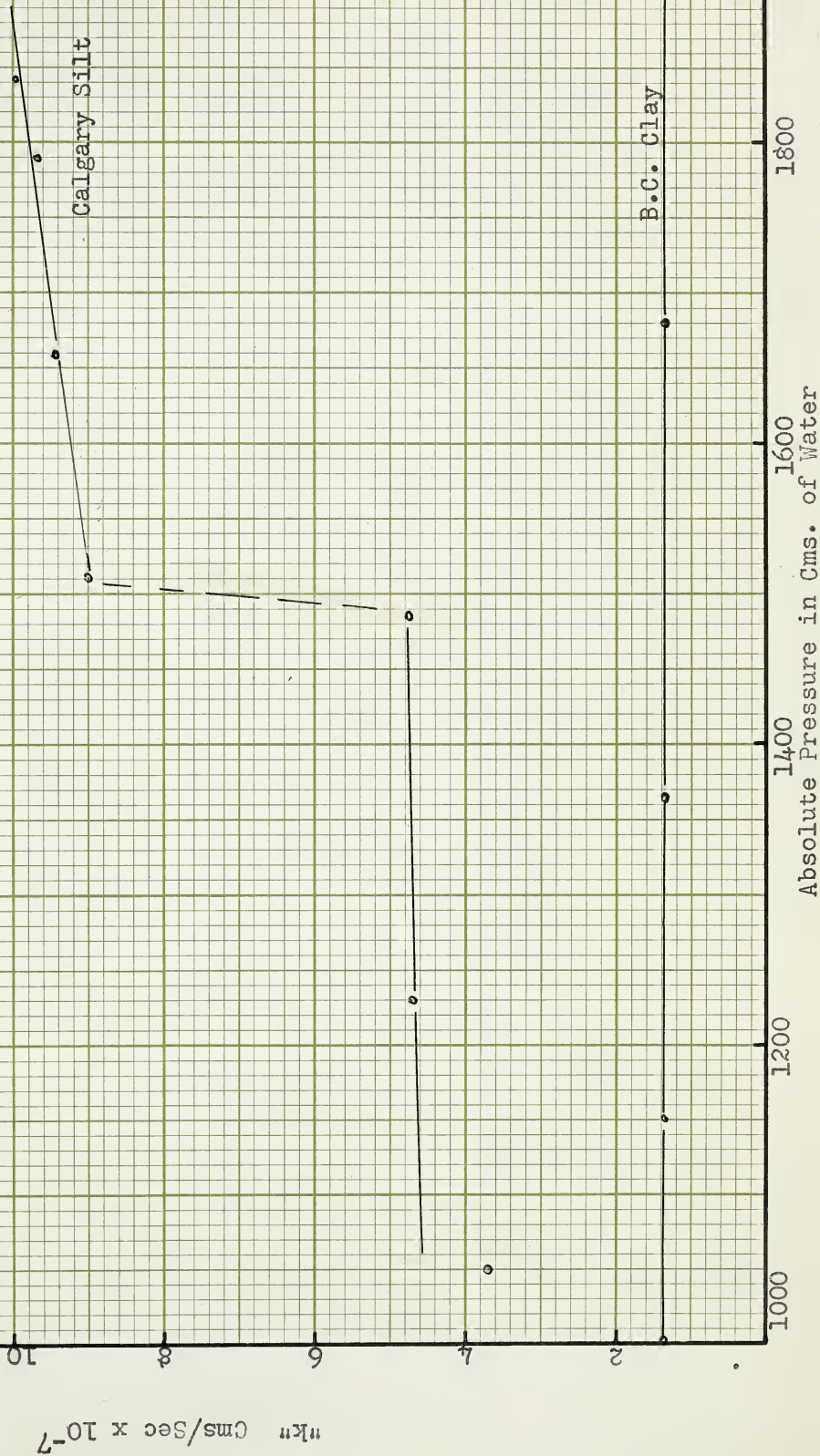
GRAPH # 4

PLOT of WATER TRANSFERRED UNDER HYDROSTATIC
HEAD in C.C. per HOUR for Calgary Silt



GRAPH # 5

PLOT of HYDRAULIC PERMEABILITY in cms per sec $\times 10^{-7}$
v.s. ABSOLUTE PRESSURE in cms of water.

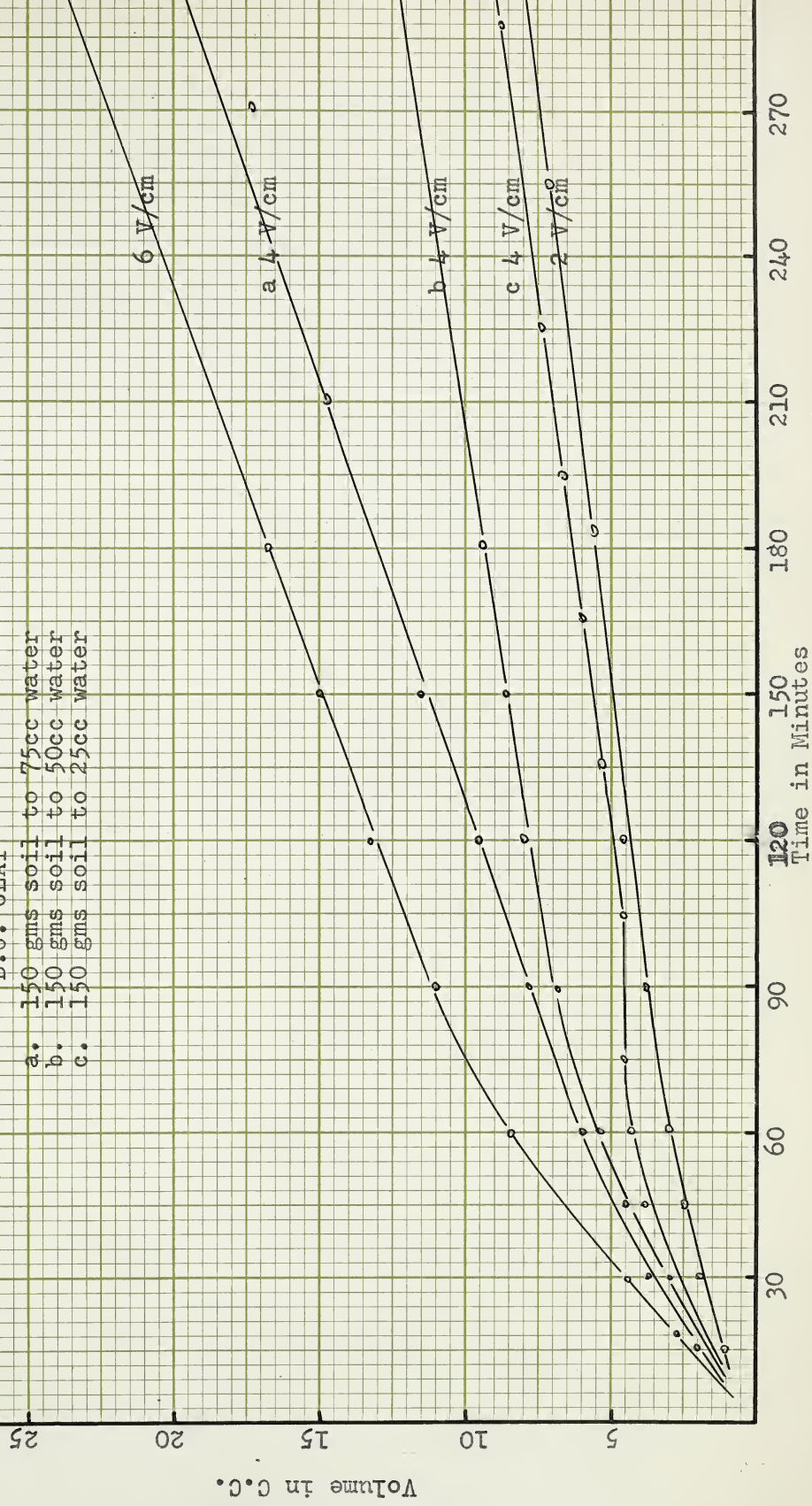


GRAPH # 6

PLOT OF VOLUME OF WATER TRANSFERRED
UNDER A POTENTIAL GRADIENT v.s. TIME

B.C. CLAY

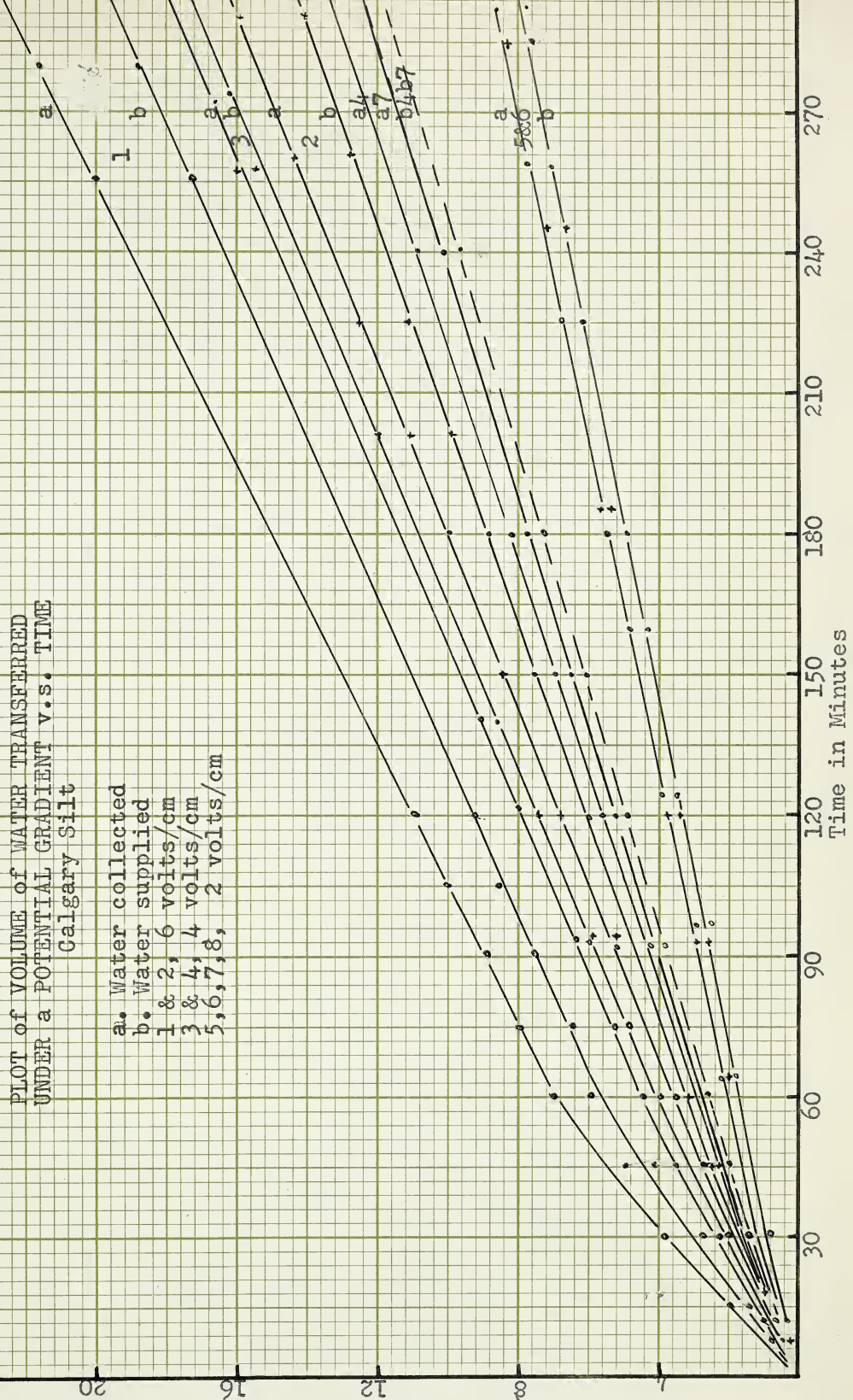
- a. 150 gms soil to 75cc water
- b. 150 gms soil to 50cc water
- c. 150 gms soil to 25cc water

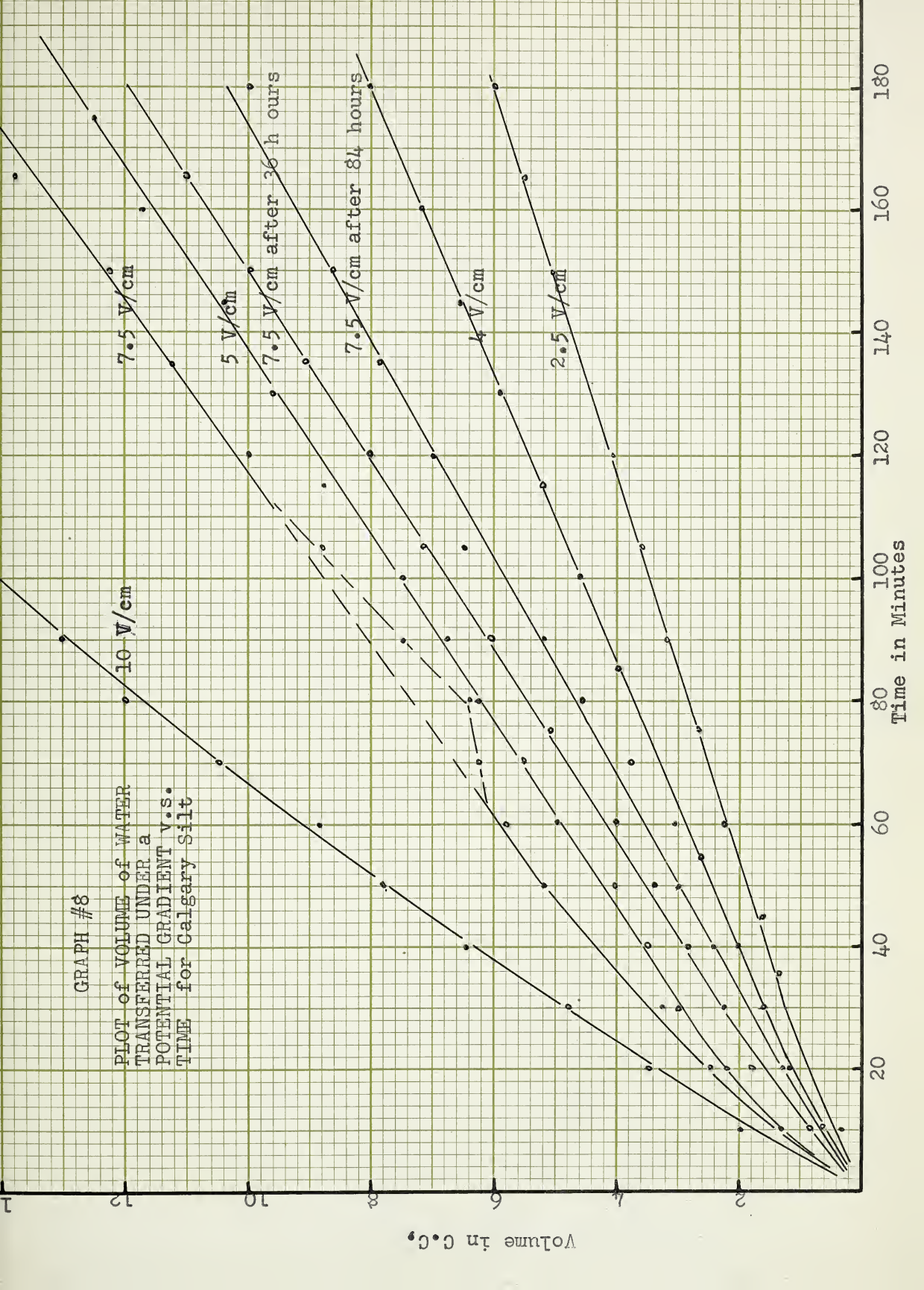


21.

Volume in C.C.

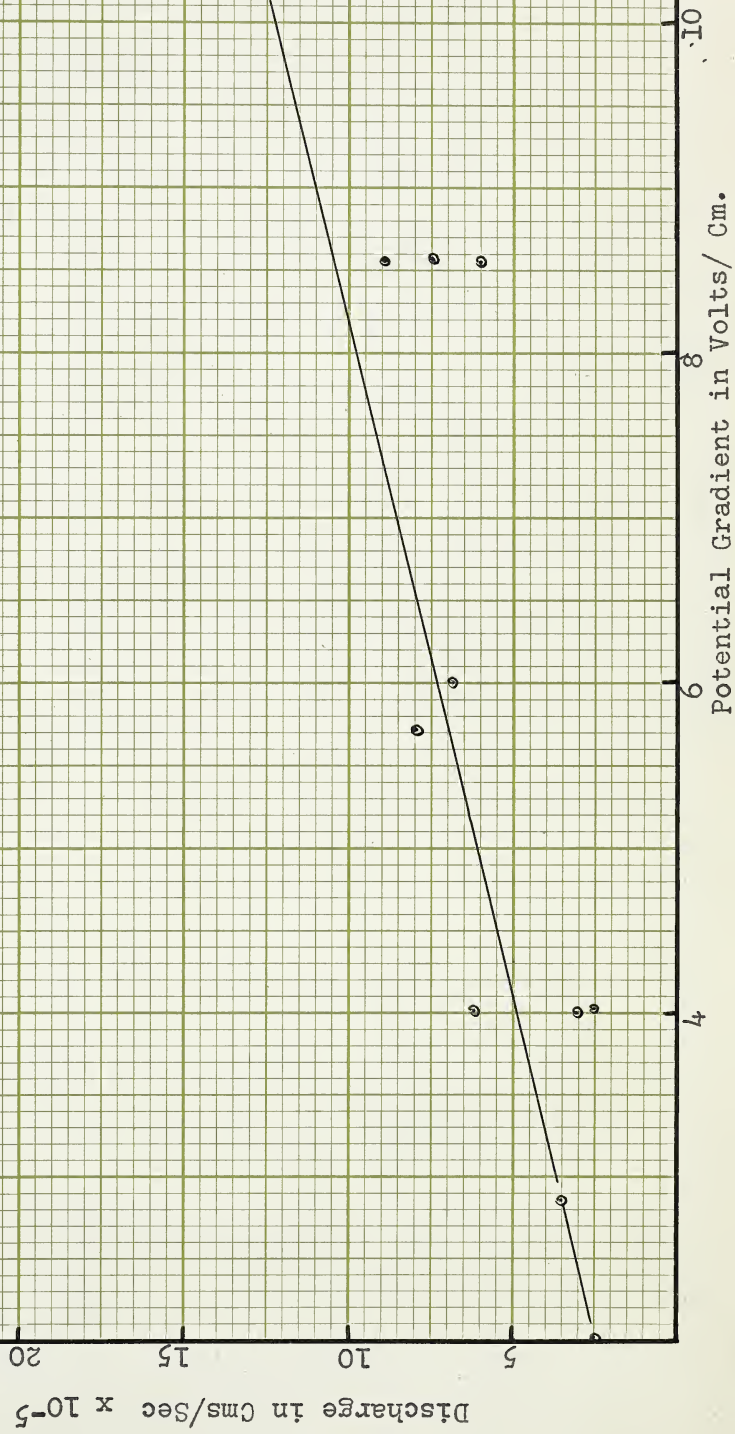
a. Water collected
b. Water supplied
1 & 2, 6 volts/cm
3 & 4, 4 volts/cm
5, 6, 7, 8, 2 volts/cm





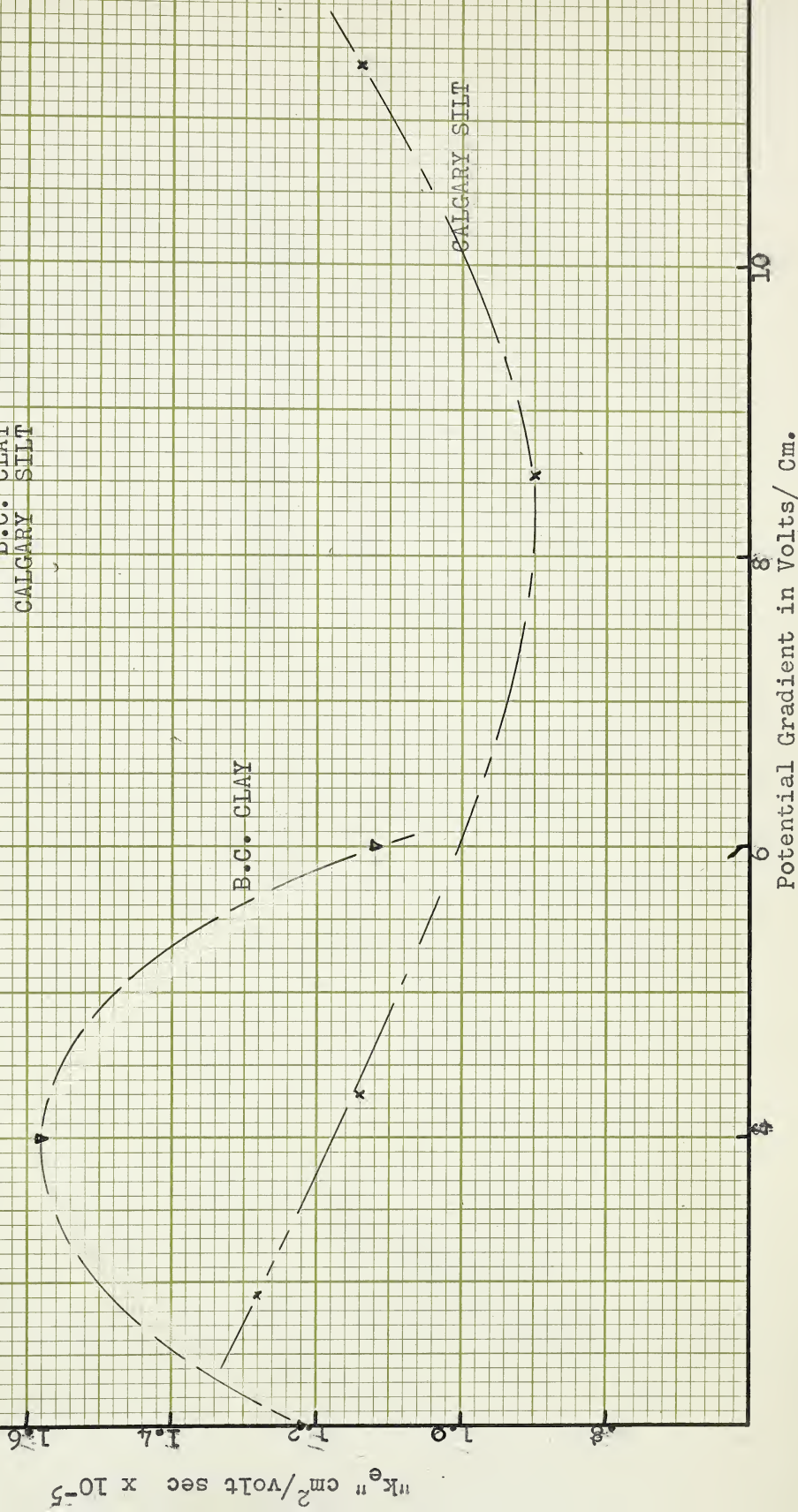
GRAPH # 9

PLOT of DISCHARGE of WATER in
CMS/Sec v.s. POTENTIAL GRADIENTS
for Calgary Silt



GRAPH 10

PLOT of ELECTRIC PERMEABILITY " k_e "
v.s. POTENTIAL GRADIENT for
B.C. CLAY
CALGARY SILT



Absolute pressure in Cms of Water

Potential Gradient in Volts/Cm.

1

2

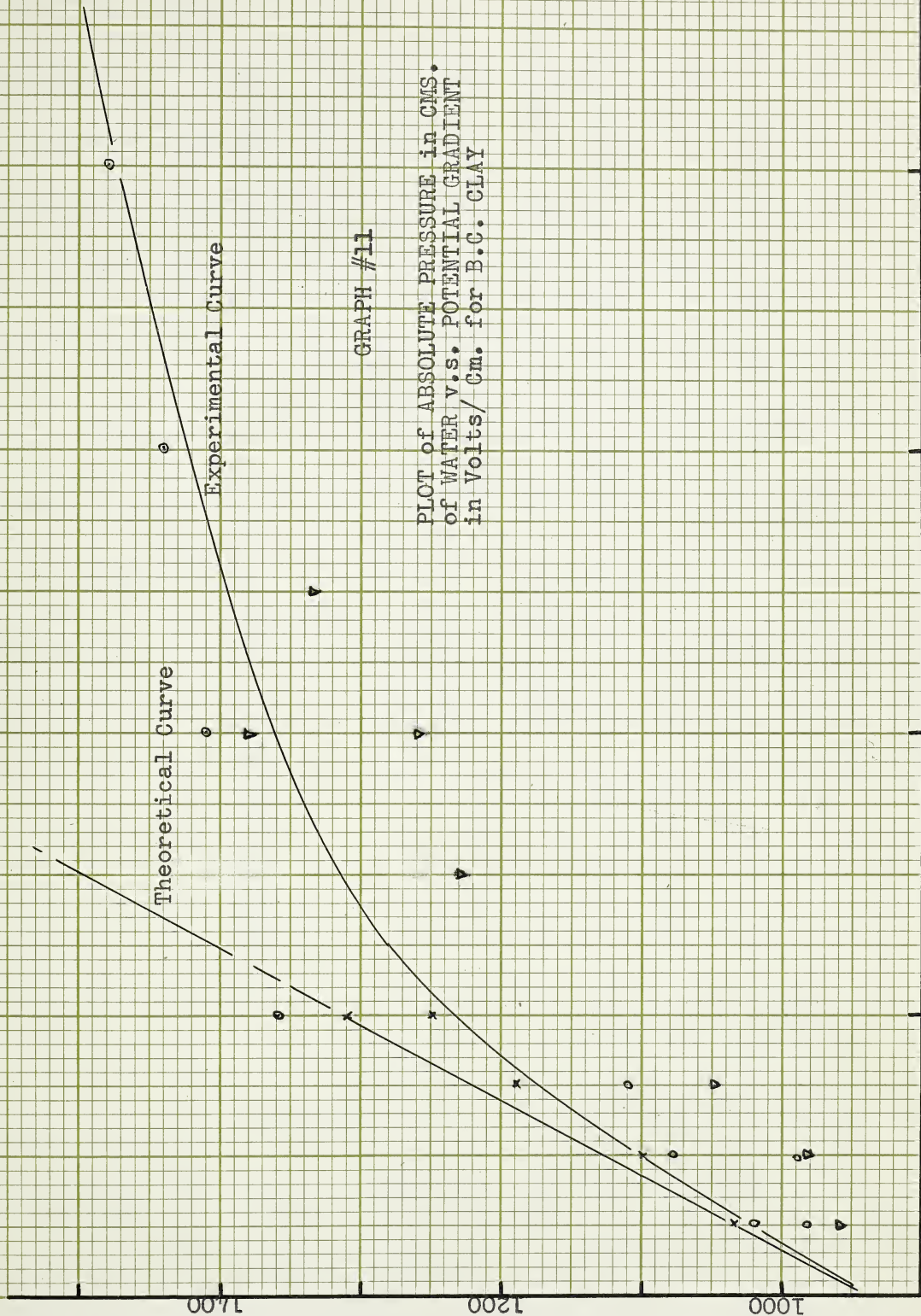
4

Theoretical Curve

Experimental Curve

GRAPH #11

PLOT OF ABSOLUTE PRESSURE in CMS.
OF WATER v.s. POTENTIAL GRADIENT
in Volts/ Cm. for B.C. CLAY



Absolute Pressure in Gms. of Water

GRAPH # 12

PLOT of ABSOLUTE PRESSURE in CMS.
of WATER v.s. POTENTIAL GRADIENT
in Volts/Cm. for Calgary Silt

Experimental Curve

Theoretical Curve

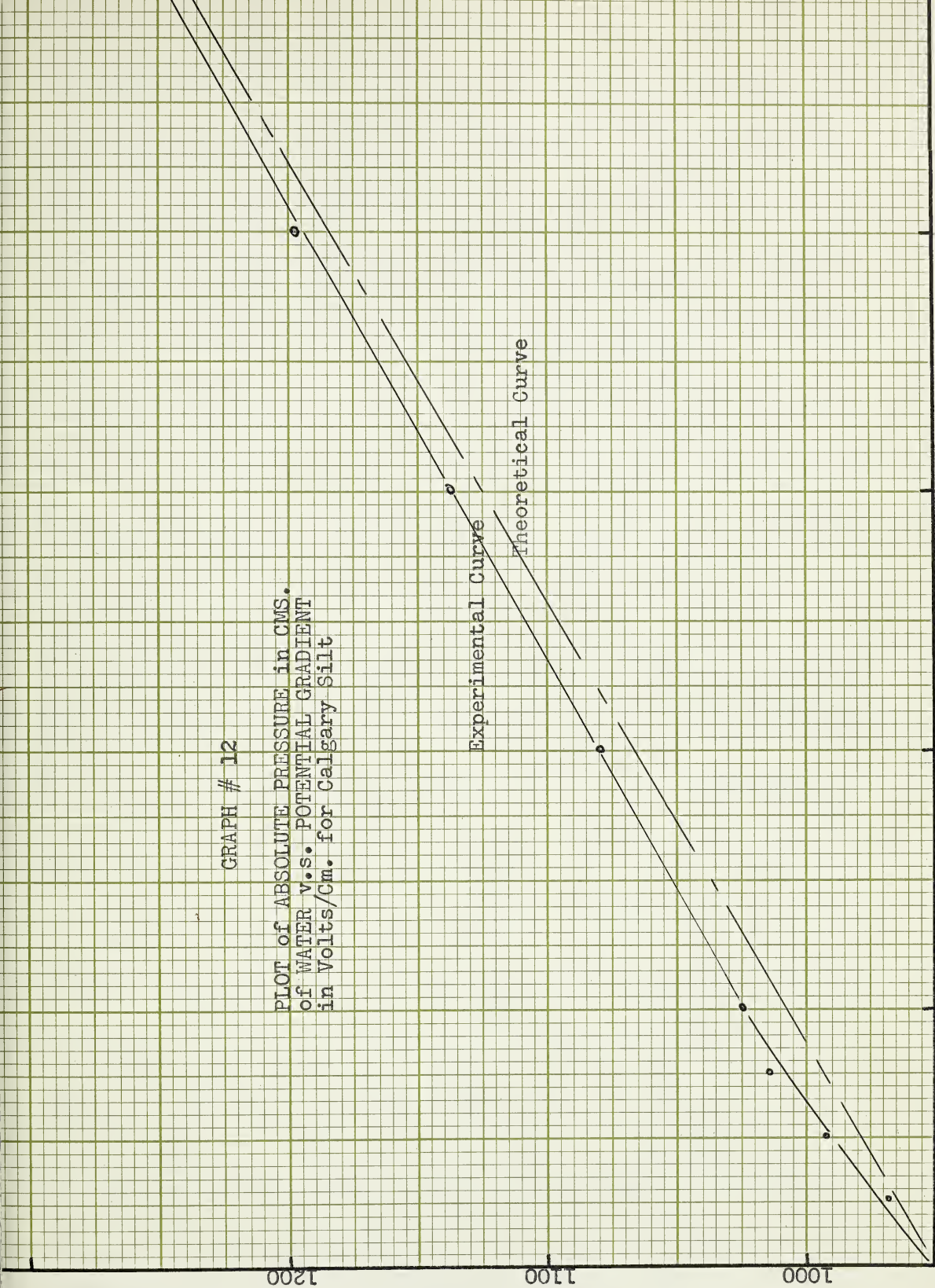
1

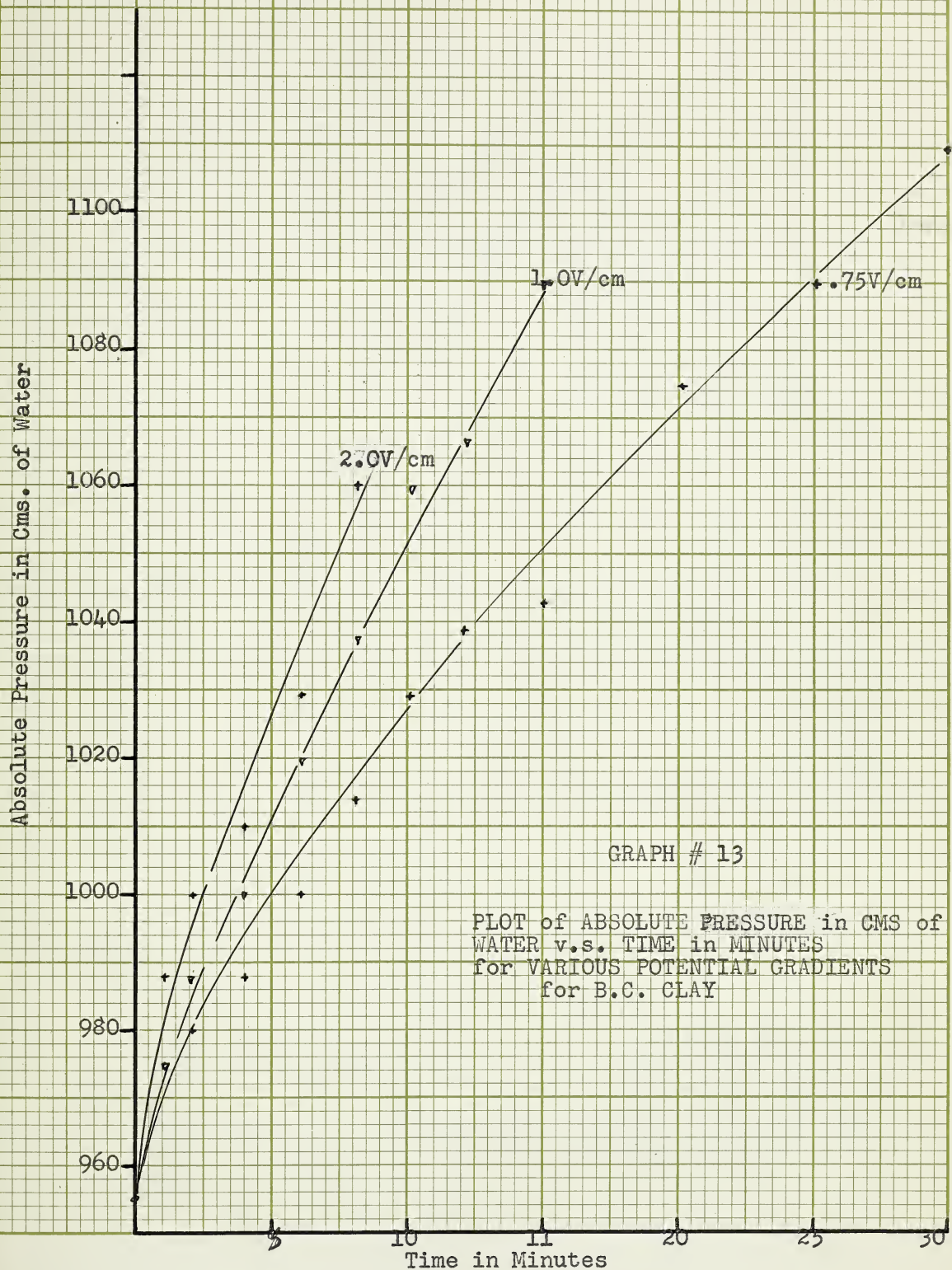
2

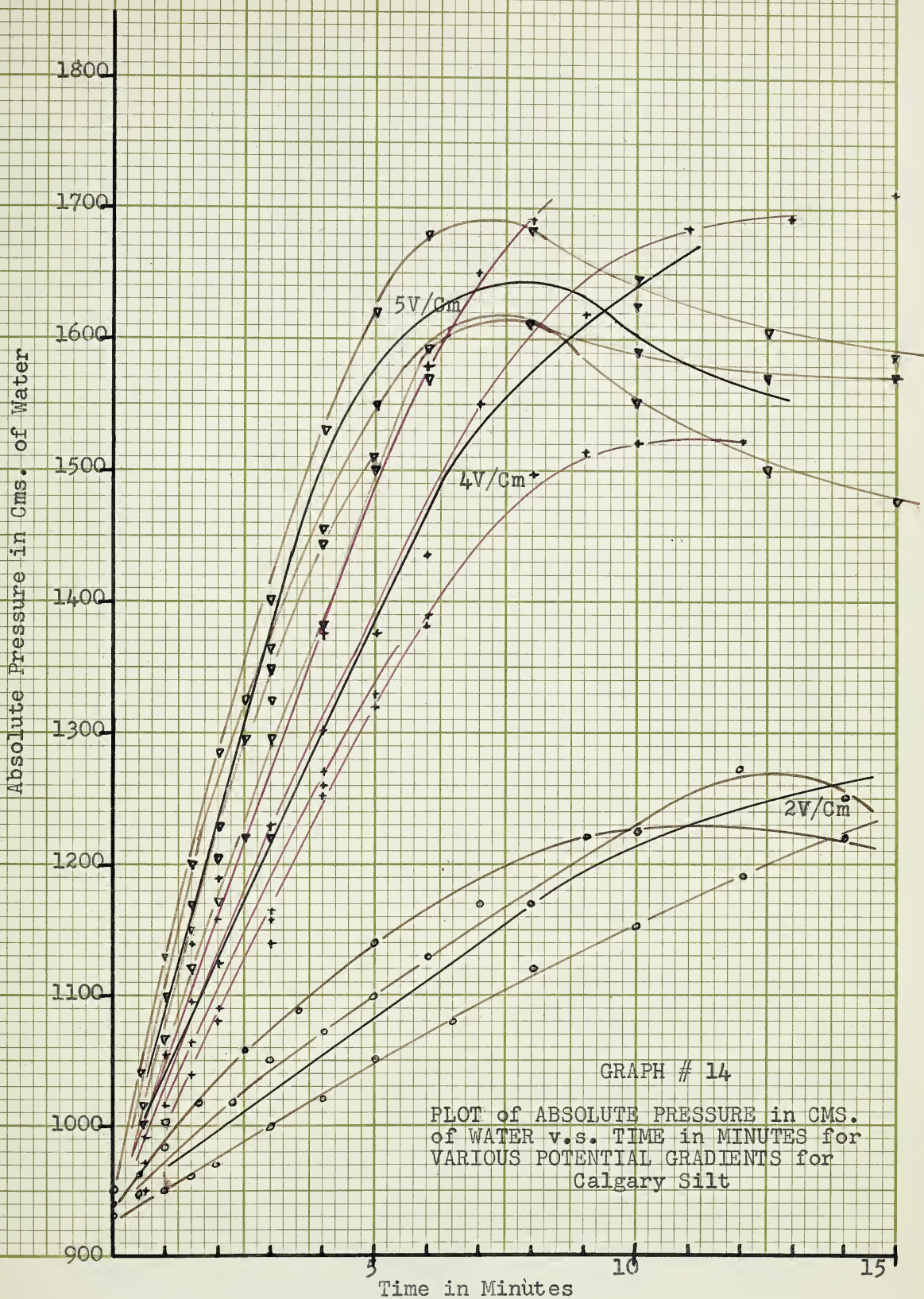
3

4

Potential Gradient in Volts/ Cm.







Absolute Pressure in Cms. of Water

GRAPH # 15

PLOT of ABSOLUTE PRESSURE in CMS.
of WATER v.s. TIME in MINUTES for
VARIOUS POTENTIAL GRADIENTS for
Calgary Silt

a...first test

b....second test on same sample

5V/Cm

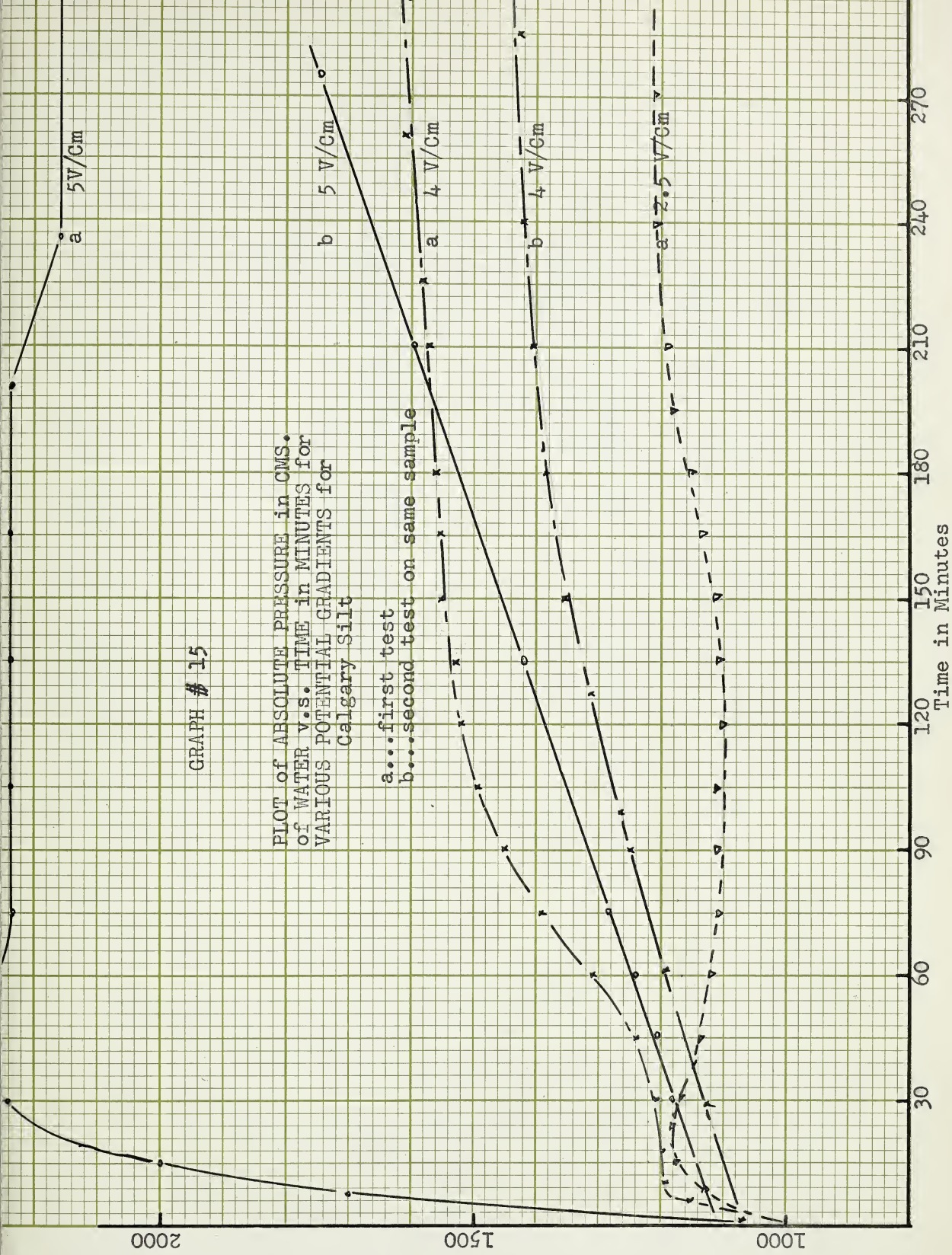
5 V/Cm

4 V/Cm

4 V/Cm

2.5 V/Cm

Time in Minutes



Current in Milliampères

GRAPH # 16

PLOT of CURRENT in Milliampères v.s.
TIME for VARIOUS POTENTIAL GRADIENTS
FOR CALGARY SILT

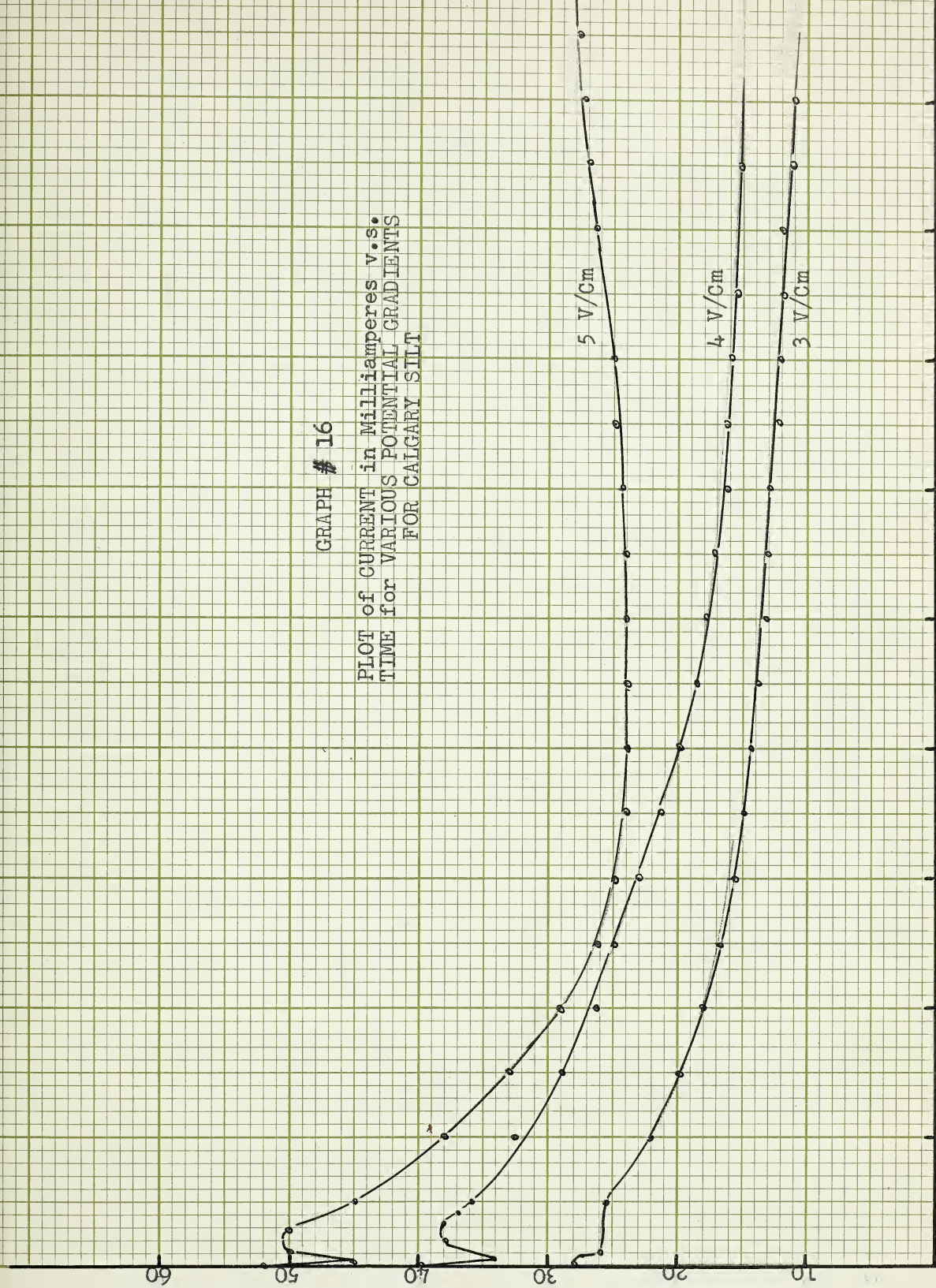
5 V/Cm

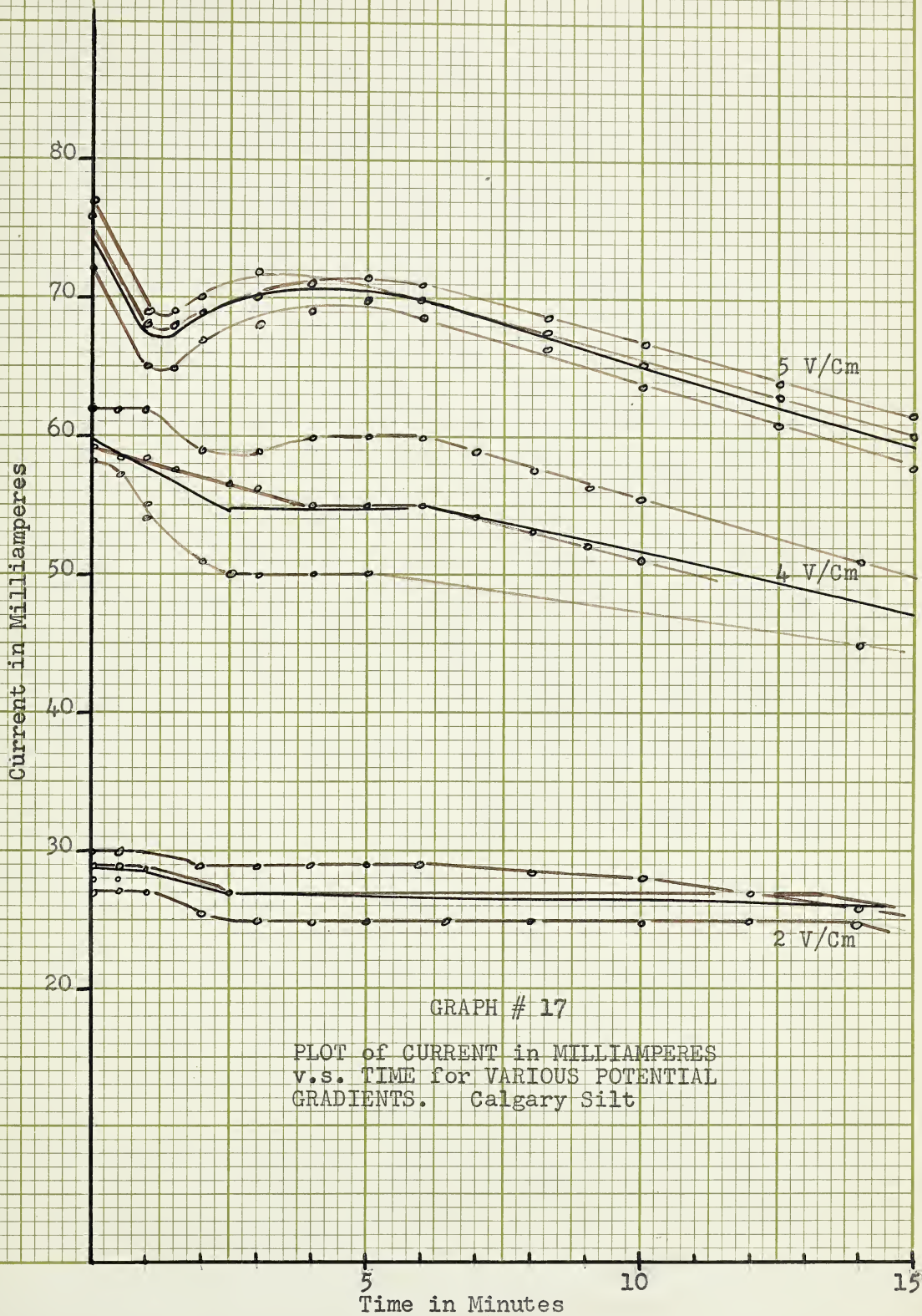
4 V/Cm

3 V/Cm

Time in Minutes

30 60 90 120 150 180 210 240 270





B29756